

DIRECT PROBLEM AND THE LIE GROUP ANALYSIS IN THE NON-LINEAR STOCHASTIC EARTH'S SURFACE SUBSIDENCE MECHANICS

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ABSTRACT: The problem is in the field of applied geo-mechanics. The investigation is focused on strata and ground movement over mined-out areas. Using the Lie group analysis of the main equation of the nonlinear stochastic geo-mechanics, a solution to determine the earth's surface subsidence is obtained. The partial differential equation of the nonlinear stochastic geo-mechanics is transformed into an ordinary one. The solution is compared with other existing solutions. The obtained relations may be used to plan and manage mining operations.

Keywords: surface subsidence mechanics, nonlinear stochastic model, Lie group analysis, direct problem

ПРИЛОЖЕНИЕ НА ГРУПОВИЯ АНАЛИЗ НА ЛИИ В МЕХАНИКА НА МУЛДАТА - ПРАВА ЗАДАЧА

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РЕЗЮМЕ: Въз основа на груповия анализ на Ли, приложен към основното уравнение на нелинейната стохастична геомеханика, е предложено решение на задачата за предвиждане на последиците от провеждането на подземни минни работи върху земната повърхност (права задача в механика на минната мулда). Анализирани са предимствата и недостатъците на разглеждания метод. Направени са изводи.

Ключови думи: механика на мулдата, групов анализ, пряка задача

Introduction

This paper is in the field of mining geo-mechanics. The investigation is focused on the mining subsidence when mining out underground ore bodies. As a basis, the nonlinear stochastic theory is proposed.

The main differential equation is obtained as a nonlinear parabolic one with the assumption that the rock mass is a stochastic medium consisting of elastic parts.

The basic goal of Lie group analysis of equations is to study the results of the application of the allowed from the equation group on the variety of its solutions.

Using the allowed from the equation group, it is possible to arrange an algebraical structure of the multitude of its solutions. This knowledge may be applied to:

- the description of the general structure of the family of all solutions of the equation;
- the determination of the types of solutions easier to be found in difference to the general solution;
- the construction of new solutions on the basis of the existing solutions, etc.

The problem

To introduce the model of the nonlinear stochastic geo-mechanics, it is assumed that the following axioms are satisfied:

- The rock mass displacements are studied from the viewpoint of the stochastic processes;
- The geo-material is stochastic medium built from elastic parts;
- The characteristics of the mining field allow the model of the new stochastic medium, introduced in (Vulkov 2006), to be applied.

By studying the plane model of cages, shown in Fig.1, and assuming that the rock mass is a stochastic medium built from elastic particles, the general equation for calculating the rock mass subsidence in the influence zone is determined in (Vulkov 1989, 2006).

On the basis of this model and according to the probability mechanisms, the following equation is obtained:

$$\left[A_{11}(P)P_x \right]_x + \left[A_{12}(P)P_x \right]_z + \left[A_{22}(P)P_z \right]_z + B_1(P)P_x + B_2(P)P_z = 0 \quad (1)$$

where $P = P(x, z)$ is the probability that a particle of the stochastic medium appears in a point with coordinates (x, z) ,

$$P_x = \frac{\partial P}{\partial x}, \quad P_z = \frac{\partial P}{\partial z}, \quad A_{11}(P), \quad A_{12}(P), \quad A_{22}(P),$$

$B_1(P)$ and $B_2(P)$ are rock mass characteristics for the subsiding process.

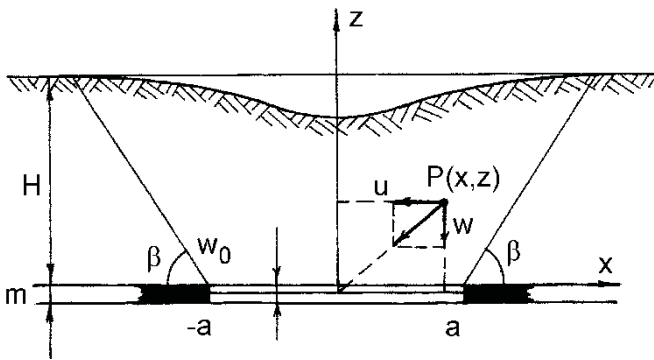


Fig. 2.

The problem for (8) – (10) can be simplified by applying the transformations (6). They transform the quasilinear parabolic partial equation into an ordinary one.

After introducing the following dimensionless variables in equation (8):

$$V = \frac{w}{w_0}; \quad A(V) = \frac{A(w)}{A(w_0)} \quad (11)$$

and substituting (6) and (11) in problems (1) – (3), it is obtained:

$$-V_{\eta} = 4[A(V)V_{\eta}]_{\eta} \quad (12)$$

$$V(0) = 0,5; \quad (13)$$

$$V(\infty) = 0, \quad (14)$$

where $V_{\eta} = \frac{dV}{d\eta}$.

To solve the studied problem, the ordinary equation (12) is integrated once:

$$V - C = -4A(V)V_{\eta}. \quad (15)$$

After separating the variables, it is obtained:

$$d\eta = -\frac{4A(V)dV}{V - C}. \quad (16)$$

Equation (16) is integrated once and the result is the function $\eta = \eta(V)$:

$$\eta = -4 \int \frac{A(V)}{V - C} dV + C_1 \quad (17)$$

where C_1 is an integration constant determined by the boundary conditions of the specific problem.

After transforming (17) in the form $V = V(\eta)$, the equation of the mining subsidence trough formed on the earth's surface is obtained.

Conclusion

In the paper, the direct problem connected with predicting the effects on the earth's surface resulting from underground mining activities is solved. This problem is reduced to Cauchy's problem for the nonlinear Fourier equation.

The partial differential equation of the nonlinear stochastic geo-mechanics is reduced into an ordinary one on the basis of the Lie group analysis of main differential equations of the nonlinear stochastic mining subsidence theory.

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