ANALYTICAL EXPRESSIONS FOR STRESSES IN A STEEPLY LAYERED ROCK MASS AROUND A CIRCULAR OPENING

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ABSTRACT. The article discusses the question of determining stresses in a steeply stratified rock mass around a circular opening. The layers are homogenous, isotropic and parallel to one another and to the axis of the opening. The thickness of layers is significantly smaller than the dimension of the hole. The influence of stresses, due to the working driving, extends into a square area. The specified class of tasks is solved by the theory of elasticity and the mechanics of the layered media. A solution of a vertical shaft, crossing the horizontal layers, is known. The stresses in each layer are expressed by the stresses in a homogenous generalized field. The characteristics of each layer and thickness are included in their expressions.
A horizontal circular opening is driven in a rock, composed of two layers. The boundary between them is inclined towards the horizon. The areas of layers are calculated when the value of the inclination angle changes from zero to ninety degrees. They are compared to their respective values at a horizontal boundary. The deviation is large, when the slope exceeds a certain limit value. This fact requires derivation of new analytical formulas for stresses.
A popular approach for determining the stresses in each layer is applied. Two group relations are used in order to obtain them. The first expresses the condition of continuity of deformations in the plane of contact between the layers. The second group includes the equilibrium of forces, expressed by generalized stresses along the layers' areas. The generalized stresses are obtained using the theory of function of complex variable. New analytical expressions for stresses are obtained. These expressions involve the areas of layers around the opening. The presented solution complements the existing expressions for stresses in horizontal layers.

Keywords: stresses, theory of function of complex variable, opening, layered media

АНАЛИТИЧНИ ИЗРАЗИ ЗА НАПРЕЖЕНИЯТА В СТРЪМНО НАПЛАСТЕН МАСИВ ОКОЛО КРЪГОВА ИЗРАБОТКА

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РЕЗЮМЕ. В статията се разглежда въпросът за определяне на напреженията в стръмно напластен масив около кръгова изработка. Пластовете са хомогенни, изотропни, успоредни помежду си и на оста на изработката. Дебелините на пластовете са значително по-малки от размерите на отвора. Влиянието на напреженията от прокарване на изработката се простира в квадратна област. Указаният клас задачи се решава с методите на теория на еластичността и на механика на напластените среди. Известно е решение за вертикална шахта, пресичаща хоризонтално разположени пластове. Напреженията във всеки пласт се изразяват чрез напреженията в еднородна обобщена среда. В техните изрази участват както характеристиките на всеки пласт така и дебелините.
Хоризонтална кръгова изработка е прокарана в масив, състоящ се от два пласта. Границата между тях е наклонена спрямо хоризонта. Изчислени са площите на пластовете, когато стойността на ъгъла на наклона варира от нула до деветдесет градуса. Те са сравнени със съответната им стойност при хоризонтална граница. Отклонението е голямо, когато наклона надвишава определена гранична стойност. Този факт изисква извеждане на нови изрази за напреженията.
Тук е приложен известен подход за определяне на напреженията във всеки стръмен пласт. За тяхното получаване се използват две групи връзки. Първата изразява условието за непрекъснатост на деформациите в равнината на контакта между пластовете. Втората група включва равновесие на усилията, изразени чрез обобщени напрежения по площите на пластовете. Обобщените напрежения са определени с метода на конформно-изобразителните функции. Получени са нови изрази за напреженията. В тези изрази участват площите на пластовете от разглежданата област. Представеното решение допълва съществуващите изрази за напреженията в хоризонтално разположени пластове.

Ключови думи: напрежения, метода на конформно-изобразителните функции, изработка, напластените среди

Introduction

Analytical and numerical methods are usually used for the calculation of stresses around openings driven in layered rock mass. The stresses around circular, rectangular and semi-circular or "d" cross section openings are examined by the finite element method (Trifonova-Genova, 2012). The rock mass consists of homogeneous layers. They are parallel to each other and to the axis of opening. The theory of elasticity in combination with the theory of layered media is applied to determine stresses around a shaft excavated into a horizontally layered rock mass (Trifonova-Genova, 2012, Sbornik ot dokladi, Varna). Analytical expressions of stresses are used for the thicknesses of layers and their physical and mechanical characteristics. This method is suitable for horizontal and slightly sloping layers.

The current article focuses on extending the scope of application. Therefore, it is necessary to study parallel layers with different inclinations and to summarize the analytical expressions of stresses in them.

Methods

1. Physical and mechanical characteristics of the generalized field

An opening in a circular form is considered. The influence of stresses due to the driving of an opening extends in a square area. It has the dimensions ($12r \times 12r$) shown in Figure 1. The rock consists of layers with thicknesses t_1 and t_2.

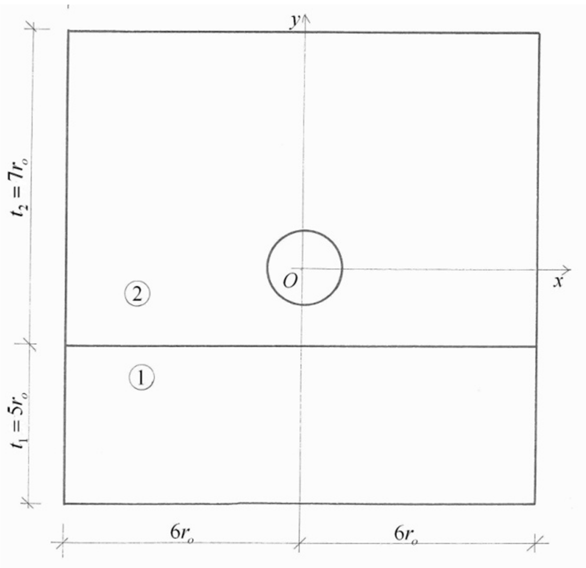

Fig.1. Horizontally layered rock

According to the theory of layered media the stresses in each layer are determined by the stresses in a homogeneous rock with the following physical-mechanical characteristics (Trifonova-Genova, 2012):

$$E^{(o)} = \frac{E^{(1)}t_1 + E^{(2)}t_2}{(t_1 + t_2)} \; ; \quad \mu^{(o)} = \frac{\mu^{(1)}t_1 + \mu^{(2)}t_2}{t_1 + t_2} . \quad (1)$$

Here $E^{(1)}$ and $E^{(2)}$ are Young's modulus for layer 1 and 2, $\mu^{(1)}$ and $\mu^{(2)}$ are Poisson's ratio for corresponding layer.

The areas of two layers are (fig.1):

$$A_1 = 12r.t_1 \; ; \quad A_2 = 12r.t_2 . \quad (2)$$

Expressions (1) can be summarized and expressed as follows:

$$E^{(o)} = \left[E^{(1)}A_1 + E^{(2)}A_2\right]A^{-1}; \quad A = A_1 + A_2;$$
$$\mu^{(o)} = \left[\mu^{(1)}A_1 + \mu^{(2)}A_2\right]A^{-1} . \quad (3)$$

Introducing Eq. (2) into Eq. (3) gives (1). This result can be summarized for n layers and applied for inclined layers:

$$E^{(o)} = \frac{\sum_{i=1}^{n} E^{(i)}A_i}{\sum_{i=1}^{n} A_i} \; ; \quad \mu^{(o)} = \frac{\sum_{i=1}^{n} \mu^{(i)}A_i}{\sum_{i=1}^{n} A_i} . \quad (4)$$

2. Influence of slope

Rock mass consists by two layers with a boundary conveying angle α to the horizon (fig. 2). This border goes off from the lower points of the circle opening.

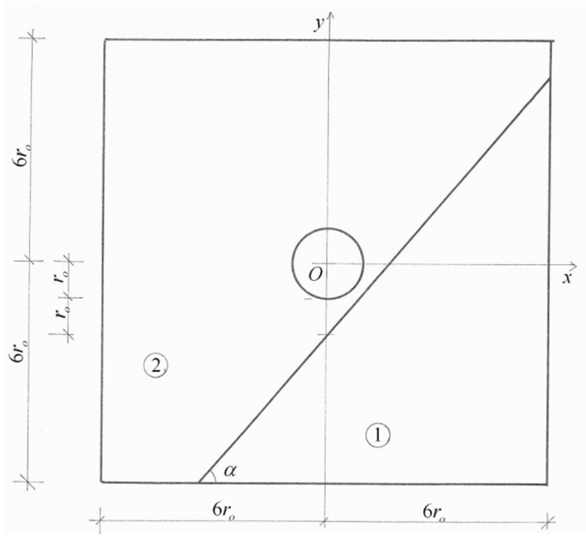

Fig.2. Inclined layered rock

The areas' values $A_{1,i}$ and $A_{2,i}$ of the layers when varying the slope α in $10°$ ($i = 1 \div 10$) are presented in Table 1. The deviations $\Delta A_{1,i}$ and $\Delta A_{2,i}$ from the respective area are obtained. The latter are calculated when the border is horizontal (Fig.1):

$$\Delta A_{1,i} = \frac{|A_{1,i} - A_{1,o}|.100}{\max\{A_{1,i}; A_{1,o}\}} [\%];$$
$$\Delta A_{2,i} = \frac{|A_{2,i} - A_{2,o}|.100}{\max\{A_{2,i}; A_{2,o}\}} [\%]. \quad (5)$$

The fourth and sixth columns of Table 1 are analyzed. It turns out that the slope's increase leads to a strong increase of indicators ($\Delta A_{1,i}, \Delta A_{2,i}$). To increase the accuracy of stresses it is necessary to apply the method described in (1) (Trifonova-Genova, 2012), but with expressions (4).

Data from Table 1 and Figure 3 show that the gradient slope at which equations (1) are used is over $45°$. Calculating the areas of the layers and applying equations (4) is recommended above this value.

Table 1.
Results

1	2	3	4	5	6
Point $i \downarrow$	α	A_1	$\Delta A_{1,i}$	A_2	$\Delta A_{2,i}$
dimension	$[^\circ]$		$[\%]$		$[\%]$
Multi plier		r^2		r^2	
1	0	60	0	84	0
2	10	60	0	84	0
3	20	60	0	84	0
4	30	60	0	84	0
5	40	60	0	84	0
6	45	60.5	0.826	83.5	0.595
7	50	62	3,17	82,0	2,34
8	60	65	14,15	78,92	6,04
9	70	67.6	11.28	76.37	9.09
10	80	70.0	7.8	74.11	11.77

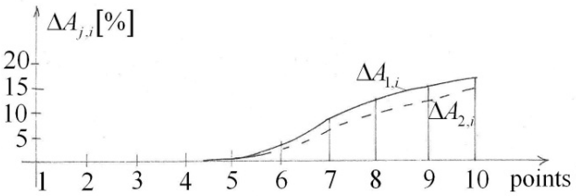

Fig.3. Diagram of deviations $\Delta A_{1,i}$ and $\Delta A_{2,i}$

2. Stresses in steep layers

The stresses at a point in each layer are given in Figure 4. The coordinate system is located along the boundary line between the two layers. The square area is loaded with vertical and horizontal in situ stresses. They are at the center of the circular hole. Vertical stresses are defined by:

$$Q_1 = \gamma_1 H \ ; \ Q_2 = \gamma_2 H . \tag{6}$$

In these expressions H is the depth of the opening, γ_1 and γ_2 is the bulk of weights of layers 1 and 2. The coefficients of the lateral pressure for horizontal stresses in the layers is determined by (fig. 4):

$$\lambda_1 = \frac{\mu^{(1)}}{1-\mu^{(1)}} ; \quad \lambda_2 = \frac{\mu^{(2)}}{1-\mu^{(2)}} . \tag{7}$$

Stresses in each layer are determined as described in (Trifonova-Genova, 2012) method. They use two groups of links. The first one expresses the conditions for equality of strains of the boundary between the layers. The second group uses the condition of equality of forces**,** expressed by generalized stresses and area of layers (Lekhnitskii, 1935; Trifonova-Genova, 2012):

$$a_{11}^{(1)}\sigma_{x'}^{(1)} - a_{11}^{(2)}\sigma_{x'}^{(2)} = \left[a_{13}^{(2)} - a_{13}^{(1)}\right]\sigma_{z'}^{(o)} ; \tag{8}$$
$$A_1\sigma_{x'}^{(1)} + A_2\sigma_{x'}^{(2)} = A\sigma_{x'}^{(o)} .$$

Here $\sigma_{x'}^{(o)}$ and $\sigma_{z'}^{(o)}$ are stresses in the generalized rock, whose expressions are given below.

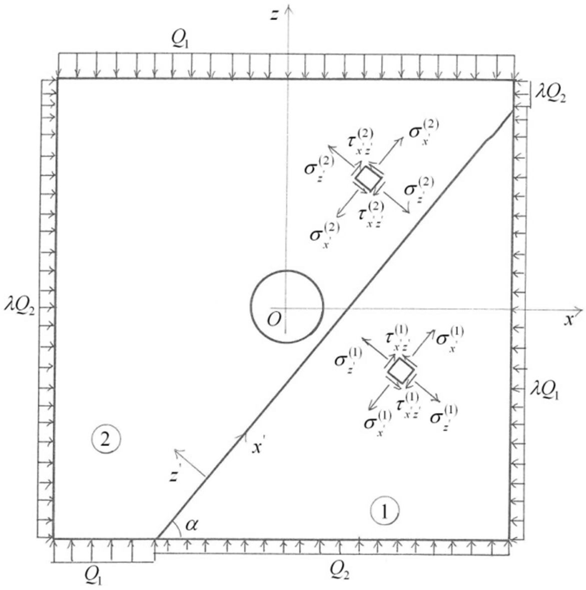

Fig. 4. Stresses in an inclined layered rock

After processing equation (8), the following stresses are obtained:

$$\sigma_{x'}^{(1)} = \frac{A_2\left[a_{13}^{(2)} - a_{13}^{(1)}\right]\sigma_{z'}^{(o)} + a_{11}^{(2)}\sigma_{x'}^{(o)}A}{a_{11}^{(1)}A_2 + a_{11}^{(2)}A_1} ;$$
$$\sigma_{x'}^{(2)} = \frac{-A_1\left[a_{13}^{(2)} - a_{13}^{(1)}\right]\sigma_{z'}^{(o)} + a_{11}^{(1)}\sigma_{x'}^{(o)}A}{a_{11}^{(1)}A_2 + a_{11}^{(2)}A_1} . \tag{9}$$

The rest of the stresses can be expressed as:

$$\sigma_{z'}^{(1)} = \sigma_{z'}^{(2)} = \sigma_{z'}^{(o)} ; \tau_{xz'}^{(1)} = \tau_{xz'}^{(2)} = \tau_{xz'}^{(o)} . \tag{10}$$

For isotropic rock the coefficients of equations (9) and (10) are expressed as follows (Trifonova-Genova, 2012; Minchev, 1960):

$$a_{11}^{(1)} = \frac{1}{E^{(1)}} \ ; \ a_{11}^{(2)} = \frac{1}{E^{(2)}} \ ;$$
$$a_{13}^{(1)} = \frac{-\mu^{(1)}}{E^{(1)}} \ ; \ a_{13}^{(2)} = \frac{-\mu^{(2)}}{E^{(2)}} . \tag{11}$$

The stresses of generalized rock in equations (9) and (10) are expressed by polar coordinates $Or\theta$ (Minchev, 1972):

$$\sigma_{x'}^{(o)} = \sigma_r^{(o)} c^2 + \sigma_\theta^{(o)} s^2 + \tau_{r\theta}^{(o)} s_2 ;$$
$$\sigma_{z'}^{(o)} = \sigma_r^{(o)} s^2 + \sigma_\theta^{(o)} c^2 - \tau_{r\theta}^{(o)} s_2 ; \qquad (12)$$
$$\tau_{x'z'}^{(o)} = \tau_{r\theta}^{(o)} c_2^2 + \left[\sigma_r^{(o)} - \sigma_\theta^{(o)}\right] 0{,}5 s_2 ,$$

where

$$c^2 = \cos^2\beta ; \quad s^2 = \sin^2\beta ; \quad s_2 = \sin(2\beta);$$
$$c_2^2 = \cos^2(2\beta); \quad \beta = \theta - \alpha ; \quad 0 \le \theta \le 90^\circ .$$

In this expression the angle θ is measured from the horizontal axis in counterclockwise direction. The stresses in the rock are given (Bulachev, 1982; Minchev, 1960):

$$\sigma_r^{(o)} = -\gamma^{(o)} H(\lambda_1 \sigma_{r2} + \lambda_2 \sigma_{r1} \cos 2\theta);$$
$$\sigma_\theta^{(o)} = -\gamma^{(o)} H(\lambda_1 \sigma_{\theta 2} - \lambda_2 \sigma_{\theta 1} \cos 2\theta); \qquad (13)$$
$$\tau_{r\theta}^{(o)} = \gamma^{(o)} H \lambda_2 \tau_{r\theta 1} \sin 2\theta ,$$

where

$$\lambda = \frac{\mu^{(o)}}{1-\mu^{(o)}} ; \quad \lambda_1 = \frac{1+\lambda}{2}; \quad \lambda_2 = \frac{1-\lambda}{2} ;$$

$$\sigma_{r2} = 1 - \frac{r_o^2}{r^2} ; \qquad \sigma_{r1} = 1 + 3\frac{r_o^4}{r^4} - 4\frac{r_o^2}{r^2} ;$$
$$\sigma_{\theta 2} = 1 + \frac{r_o^2}{r^2} ; \qquad \sigma_{\theta 1} = 1 + 3\frac{r_o^4}{r^4} ;$$
$$\tau_{r\theta 1} = 1 - 3\frac{r_o^4}{r^4} + 2\frac{r_o^4}{r^2} ; \qquad \gamma^{(o)} = \frac{\gamma^{(1)} A_1 + \gamma^{(2)} A_2}{A} .$$

Here r_o is the radius of opening (Fig.4). The stresses of equation (13) are obtained by complex variable theory (Muskhelishvili, 1963; Savin, 1961; Bulachev, 1982). The stresses in the contour of opening in generalized rock are of practical interest:

$$\sigma_r^{(o)} = 0 ; \ \tau_{r\theta}^{(o)} = 0;$$
$$\sigma_\theta^{(o)} = -2\gamma^{(o)} H(\lambda_1 - 2\lambda_2 \cos 2\theta). \qquad (14)$$

3. Key finding

Analytical expressions of stresses in each layer are applied in layers whose slope is greater than half the right angle.

These expressions are summary of the last expressions of stresses in layers (Trifonova-Genova, 2012). The latter are used in horizontal and parallel layers. The thicknesses of layers are included in them.

Conclusion

The method, described in this article has the following advantages:
-it is very simple to be implemented;
-it can also be used in transversal-isotropic layers (Lekhnitskii, 1935);
-it can be summed up for many layers;
- It can be used in fixed opening, driven at great depth (Minchev, 1960; Bulachev, 1982);
- It can be used taking into account the pitch of the terrain (Li et al., 2008).

The results obtained are applicable in the design of mining works. They are particularly suitable for close physical and-mechanical characteristics. Here we mean the relation between the maximum and minimum values of Young's modulus for layer 1 and 2.

References

Лехницкий С. Г., К расчету на прочность составных балок, Вестник инженеров и техников, 1935, 9. (Lekhnitskii S. G., K raschetu na prochnost sostavnih balok, Vestnik inzhenerov i tehnikov, 1935, 9.)

Минчев И. Тр. Динамична теория на еластичността, Д.И. „Техника“, С., 1972, 285с. (Minchev I. Tr., Dinamichna teoriya na elastichnostta, D. I. “Tehnika”, S., 1972, 285s.)

Минчев И. Тр., Механика на анизотропното тяло в минното дело, Д.И. „Техника“, С., 1960, 126с. (Minchev I. Tr., Mehanika na anizotropnoto tyalo v minnoto delo, D. I. “Tehnika”, S., 1962, 126s.)

Трифонова-Генова В.М. Деформирано състояние на разномодулен масив в околността на вертикална изработка, *V-та Международна научна конференция по геомеханика, Сборник от доклади, Варна*, 18 - 20.06.2012, 294-299с. (Trifonova-Genova V. M., Deformirano sastoyanie na raznomodulen masiv v okolnostta na vertikalna izrabotka”, *V-ta Mezhdunarodna nauchna konferentsia po geomehanika, Sbornik ot dokladi, Varna,* 18-20.06.2012, 294-299s.)

Трифонова-Генова В. М., Напрегнато състояние в напластен масив около кръгла изработка, София: ИК “Св. Иван Рилски”, 2012.-132с. (Trifonova-Genova V. M., Napregnato sastoyanie v naplasten masiv okolo krugla izrabotka, Sofia: I. K. “Sv. Ivan Rilski”, 2012.-132s.)

Булычев Н. С., Механика подземных сооружений, М., Недра, 1982, 282с. (Bulachev N. S., Mehanika podzemnih sooruzhenii, M., Nedra, 1982, 282s.)

Lekhnitskii S. G., Theory of elasticity of an anisotropic elastic body. San Francisco: Holden-day; 1963.

Muskhelishvili N. I., Some basic problems of mathematical Theory of elasticity. Groningen: Noordhoff; 1963.

Savin G. N. Stress concentration around holes. London: Pergamon; 1961.

Shu-cai Li, Ming-bin Wang, An elastic stress-displacement solution for lined tunnel at great depth, Int. J. of Rock Mech. & Min. Sci. 45 (2008) 486-494.

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