# University of Mining and Geology
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# EXTENDED SUMMARY

### of Doctoral (PhD) Dissertation

# GEODETIC EARTH OBSERVATIONS COMBINATION FOR VERTICAL REFERENCE SYSTEM DEFINITION – A CASE STUDY FOR THE KINGDOM OF SAUDI ARABIA VERTICAL DATUM AND VERTICAL REFERENCE FRAME


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**March, 2020**

**SUBMISSION STATEMENT:**

The dissertation titled

‘Geodetic Earth Observations Combination for Vertical Reference System Definition – A Case Study for the Kingdom of Saudi Arabia Vertical Datum and Vertical Reference Frame’

is presented for examination in partial fulfillment of the requirements for a doctoral degree (PhD) at the UNIVERSITY OF MINING AND GEOLOGY "ST. IVAN RILSKI", on December 13, 2019, and approved for public presentation.

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Prof. Dr. Momchil Minchev & Dr. Rossen Grebenitcharsky

March 12, 2020

Sofia, Bulgaria

This extended summary is prepared to present the full thesis in an extended summary emphasizing its most important details and contributions. Only selected sections, sub-sections and paragraphs are compiled in their original form as separate citations. All chapters, sections and sub-sections together with equations, figures and tables are presented with their original numbers in order to make easier the reference to those in the original thesis text.

# Chapter I: Introduction

This chapter presents the current status, the reliability and the importance of the problem of combination of different types of Geodetic Earth Observations (GEOs) for Vertical Datum (VD) and Vertical Reference System (VRFS) definition both worldwide (examples for vertical datum definitions in the past) and in the Kingdom of Saudi Arabia (KSA). Also, the thesis research tasks and research proposal for solutions have been discussed in terms of problem statement; research aims and objectives; applied methods and the structure of the thesis itself.

## *A. Introduction and research problem statement*

### 1. Current status of the problem of combination of different types of Geodetic Earth Observations for Vertical Datum and Vertical Reference System definition

#### 1.1. Worldwide

Due to long time period of historical data having different nature, different accuracy, resolution and etc. currently derived orthometric heights of tide-gauges (TGs) utilized for local and regional vertical reference frame (VRF) definition do no coincide each other or have big discrepancies along the coastlines even over small territories or islands. Apart of such problems those orthometric heights show great discrepancies with regard to those derived from least-squares adjustments (LSA) of precise leveling networks. These problems would cause distortions in the precise leveling networks, if certain TG orthometric heights (derived from geoid models) are going to be fixed during the LSA procedure. Such examples still exist for old historical leveling networks in USA, Canada, Australia, European Vertical Reference Frame (EVRF), KSA Jeddah'69 in Saudi Arabia and etc.

#### 1.1.1.Examples for vertical datum definitions in the past

**THE USA:**

**National Geodetic Vertical Datum of 1929 (NGVD 29):** *The datum was not mean sea level, the geoid, or any other equipotential surface. Therefore, it was renamed in 1973, the National Geodetic Vertical Datum of 1929*.

This example is considered as the first multi TG vertical datum but referred to fixed Mean Sea Level at different TG stations which would be affected by the Dynamic Ocean Topography (DOT)/Sea Surface Topography (SST).

**North American Vertical Datum (NAVD):** For the new general adjustment of the North American Vertical Datum of 1988 (NAVD 88) (Zilkoski, D., Richards, J.,Young, G. , 1992, a minimum-constraint adjustment of Canadian-Mexican-U.S. leveling observations was performed holding fixed the height of the primary tidal benchmark, referenced to the new International Great Lakes Datum of 1985 (IGLD 85) local mean sea level height value, at Father Point/Rimouski, Quebec, Canada. *It means that the NAVD was defined by the classical 1D TG record analysis related to International Great Lakes Datum of 1985*.

Preliminary analyses as per the same reference above indicate differences for the conterminous United States between orthometric heights referred to NAVD 88 and to the National Geodetic Vertical Datum of 1929 (NGVD 29) ranging from -40 cm to +150 cm. In Alaska, the differences range from +94 cm to +240 cm. Those differences show the effect of DOT/SST per MSL and the magnitude of deviations with regard to physical heights referenced to an equipotential surface (local or regional geoid).

As shown below, *in 2022, the US National Geodetic Survey (NGS) will replace the North American Vertical Datum of 1988 (NAVD 88) with a geoid-based height reference system called the North American-Pacific Geopotential Datum of 2022 (NAPGD2022).*

**CANADA:**

Being part of computations for both NGVD29 and NAVD88, Canadian geodetic authorities decided to return back to NGVD29 due to great discrepancies with regard to NAVD88 under conditions they would work on the definition of new Canadian 3D vertical datum based on new Canadian geoid.

**Canadian Geodetic Vertical Datum of 2013 (CGVD2013):** Natural Resources Canada (NRCan) has released the Canadian Geodetic Vertical Datum of 2013 (CGVD2013), which is now the new reference standard for heights across Canada. This height reference system replaced the Canadian Geodetic Vertical Datum of 1928 (CGVD28).

CGVD2013 is defined by the equipotential surface ($W_0$=62,636,856.0 $m^2s^{-2}$), which represents by convention the coastal mean sea level for North America. This definition comes from an agreement between the United States of America and Canada. This new vertical datum is realized currently by the geoid model CGG2013a, which provides the separation between the GRS80 ellipsoid and the above described surface in the NAD83(CSRS) reference frame, making it compatible with global navigation satellite systems (GNSS) such as GPS.

*The future US datum North American-Pacific Geopotential Datum of 2022 (NAPGD2022).Would have the same definition as CGVD2013 and at the time of release **will enable a unified continental height system***.

## EUROPEAN UNION (EU):

**European Vertical Reference Frame 2007 (EVRF2007):** According to BKG EVRS - EVRF2007 Report (2007), the EVRF2007 results are adopted by EU in 2008 in Brussels and distributed by the end of 2008. It has the following specifications:

In the Unified European Leveling Network 95/98 (UELN95/98) solution that forms the backbone of the EVRF2000, the Normaal Amsterdams Peil (NAP) mean sea level was fixed by the reference point 000A2530 in the Netherlands.

Therefore, the UELN adjustment for the EVRF2007 is fitted to the EVRF2000 solution by choosing a number of datum points and introducing their UELN95/98 heights into the free adjustment of the current network. For this transformation purpose it was important to choose stable marked datum points located in a stable part of the European plate. These datum points are part of both adjustments 2007 and 1998. The participating countries located in the stable part of the European plate were asked to propose datum points for the EVRF2007. Finally, 13 datum points were used.

*At the end, EVRF2007 is linked to the NAP TG station through: 1) Point 000A2530 and 2) 13 datum points fixed to their UELN95/98 heights linked to the vertical datum of EVRF2000. Or, it can be considered as a classical 1D vertical datum by one TG station.*

**New European Vertical Reference Frame 2019 (EVRF2019):** EVRF2019 (Sacher, 2019) is a new realization of EVRS. The vertical datum is defined by 13 datum points with their geopotential numbers of EVRF2007 and: 1) New selection of 13 datum points widely distributed, 1 point per country; 2) no datum points in area of vertical land movements; 3) no datum points in countries with known systematic tilts or big differences to the former realization.

*At the end, EVRF2019 is linked again to the NAP TG station through the geopotential numbers of new 13 datum points in EVRF2007.*

## AUSTRALIA:

**The Australian Height Datum 1971 (AHD71):** According to Geoscience Australia (2019a,b,c) AHD71 is the official national vertical datum for Australia and refers to Australian Height Datum 1971. AHD was adopted by the National Mapping Council in May 1971 as the datum to which all vertical control for mapping was to be referred. The datum surface pastes through approximate mean sea level (MSL) realized between 1966 and 1968 at tide gauges around the coastline. It has the following specifications: The interconnected network of level sections and junction points was constrained at the 30 tide gauge sites, which were assigned a **value of zero AHD**. The least squares adjustment propagated mean sea level heights, or AHD, across the level network. Despite the best efforts of surveyors, systematic, gross and random errors crept into the level sections and were distributed across the network within the least squares adjustment (LSA). This is similar example to NGVD 29

**New Geocentric Datum of Australia 2020 (GDA2020):** According to Geoscience Australia (2019b) Australia's datum will be modernized in a two-stage approach (static and kinematic) to ensure that Australians continue to have access to the most accurate location information. The vertical component of GDA2020 is realized as **Australian Geoid 2020 (AUSGeoid2020):** According to Geoscience Australia (2019c) it is Australian Vertical Datum (2020) in the form of 3D Geoid.

*A detailed summary of historical development of Vertical Datum concept can be found in the thesis and a very general abstract is presented in Fig. 2.26. It shows the evolution of the idea for vertical geodetic datum and support the significance of the main problem statement from scientific point of view and from pure practical/application point of view for the KSA case study.*

**SHORT OVERVIEW OF EXISTING AND FUTURE VERTICAL DATUMS**

**IN THE PAST:**

**VERTICAL DATUM - LOCAL**

- **One point – BM linked to a Tide Gauge with averaged Mean Sea Level using hourly TG data for several years**

  **- Differences between LOCAL DATUMS due to Sea Surface Topography (SST) /Dynamic Ocean Topography (DOT) up to 1-1.5 meters. Differences between vertical datums of different countries**

- **Several TGs are used and an average MSL is computed from those TG data (like North American Datum 1983 – NAD 83)**

  **– Does not solve the problem with SST (DOT)**

**NOW: DIFFERENT SURFACES ARE USED AS VERTICAL DATUMS**

- **Ellipsoid for ellipsoidal heights (GPS) – pure geometrical datum**
- **1) Quasi-geoid for NORMAL HEIGHTS for the Earth Surface**
  **2) Ellipsoid as vertical datum for the TELLUROID (Semi-Earth Surface) using NORMAL HEIGHTS**
- **VERY PRECISE GEOID as VERTICAL DATUM FORTHOMETRIC OR DYNAMIC HEIGHTS but still SST (DOT) need to be accurately known**
- **GLOBAL PRECISE VERTICAL DATUM – Global geoid from GOCE**
  - **TO UNIFY THE LOCAL VERTICAL DATUMS DEFINED BY TGs AFTER PRECISE SST VALUES (BASED ON GOCE DATA) ARE SUBSTRICTED FROM TG DATA**

**FUTURE:**

- **VERY PRECISE LOCAL (REGIONAL GEOIDS) USED AS VERTICAL DATUMS REFERENCED TO A GLOBAL VERTICAL DATUM**
- **TIME VARYING GEOID AS A DYNAMIC VERTICAL DATUM - VERTICAL DATUM CHANGING IN TIME**

Figure 2.26. Overview of vertical datums utilized in the past, now days and future realizations

For most classically defined VDs great discrepancies exist with regard to newly defined local and regional geoids. At the same time, parallel to classical definitions of VD many additional different types of geodetic Earth observations already exist at TG Networks (see Chapter IV). Their combination should be utilized in order to provide a reliable compatibility between precise leveling & TG derived VRFs in terms of geo-potential values/numbers and those VRFs defined by gravimetric geoid models.

### 1.2. Kingdom of Saudi Arabia

The existing Geodetic Leveling Network (GLN) of the Kingdom of Saudi Arabia is now more than 50 years old. It was established between 1966 and 1971. After several re-processing attempts by ARAMCO in 1971 and in 1978, a great difference between leveling height of TGBM in Ras-Tanura and the actual height above MSL at the TG station was found. It was confirmed to be -79 cm and for many years was considered to be due to difference in MSLs in Jeddah and RasTanura.

Unfortunately, 85 percent of the network had been destroyed in time leaving the country with inadequate vertical control of 383 BMs irregularly distributed over the Kingdom. The level lines generally follow the roads due to ease of access. The majority of the network has been destroyed during the rapid infrastructure development that has taken place during last decades.

In 2009-2010, the General Commission for Survey aimed to re-establish the GLN with Second Order / Class I accuracies and density of original network by the installation, measurement and computation of new replacement stations (based on existing GLN stations wherever they still exist). The project takes into account the regeneration of the GLN to its original point density and quality, suitable to serve as the basis for future development within the Kingdom.

After completing all precise leveling observations and absolute & relative gravity observations in 2017 GCS has decided to establish a new Vertical Reference Frame (VRF) based on Least-Squares Adjustment (LSA) by fixing the geo-potential number/orthometric height with regard to Mean Sea Level of Tide Gauge TG) Jeddah Bench Mark B. The geopotential number and orthometric height of TGBM-B have been determined after analysis of 3.5 years TG records together with ~ 30 years of Satellite Altimetry and utilizing the most recent Global Gravity Models (GGMs). In 2018, the new Kingdom of Saudi Arabia Vertical Reference Frame Jeddah 2014 (KSA-VRF14) has been officially announced and started to be implemented by GCS, KSA.

As result of the complex study, the DOT/SST differences between Jeddah and Ras Tanura MSLs are found to be around 10 cm and the difference between LSA orthometric height in the new VRF and the height of TGBM in Ras-Tanura is still in order of ~ 30cm (containing the DOT/SST shifts in MSLs). Despite there is a great reduction of the 79 cm down to ~20 cm in height differences between the MSL in Jeddah and those in Ras Tanura such discrepancy needs to be explained and confirmed.

For this reason, the current study has been triggered, considering the fact that many different types of Geodetic Earth Observations (GEO) exist and are collected in a relatively short time with very high accuracy requirements. They can be utilized in order to ***improve*** the KSA-VRF14 by determination of geopotential shifts per every KSA-NTGN station and utilizing them by a constrained LSA of the entire KSA National Vertical Network (KSA-NVN).

## 2.Main research tasks and research proposal for their solutions

Nowadays, the impact of Spatial Reference Systems in everyday activities is increasing for global, regional, national and local applications. All new remote sensing and Earth observations require a geo-referencing to Spatial Reference Systems for different applications and coverage. The Kingdom of Saudi Arabia has initiated the definition, establishment, maintenance and dissemination of its own Saudi Arabia National Spatial Reference System (SANSRS). SANSRS consists of two major components: 1) An unified Kingdom of Saudi Arabia Geodetic Reference Frame (KSA-GRF) and 2) Kingdom of Saudi Arabia Vertical Reference Frame (KSA-VRF) incorporating a Vertical Datum (VD). More information about Kingdom of Saudi Arabia National Geodetic Infrastructure (KSA-NGI) can be found in Chapter IV.

### 2.1.Problem statement

- **BRIEF OVERVIEW:** Having many different types of collocated geodetic observations at a TG station would allow the computation of geopotential values and numbers and the orthometric heights of a BM from the TG Network to be determined up to the highest possible accuracy requirements.

- **DATA AND INFORMATION AVAILABLE:** The following types of data are available/going to be available at KSA National Tide Gauge Network (KSA-NTGN): 1) 7.5 years of TG records at 12 TG stations along Read Sea and Arabian Gulf coastal line; 2) precise leveling data for monitoring stability of every TG Network since 2014; 3) Orthometric heights above MSL in KSA-VRF14; 4) Precise gravity values for main TGBMs for every TG stations; 5) Geoid Values at TGBMs from current gravimetric Geoid GRA-GEOID17 and the hybrid geoid KSA-GEOID17; 6) GGM geoid values for most recent Earth GGMs for TGBMs for every TG station; 7) Dynamic Ocean Topography/Sea Surface Topography values for TGBMs from the most recent models; 8) Dynamic Ocean Topography/Sea Surface Topography values for TGBMs based on analysis of ~34 years all available Satellite Altimetric Missions for Read Sea and Arabian Golf; 9) Cartesian 3D, Geographical Coordinates and Ellipsoidal heights for main TGBMs in: the National Geodetic Reference Frame (KSA-GRF17); 10) Continuous Reference Stations (CORS) ellipsoidal height time series in the vicinity of TG Networks (up to several km distance); 11) Gravity data coverage around TG stations from airborne gravity.

- **OPEN RESEARCH QUESTIONS/HYPOTHESES:** The following Research Hypothesis (RHs) could be assumed during the execution of this research proposal:
    - RH 1: From theoretical point of view the combination of different data and information sources from different collocated geodetic earth observations per a TG station and its vicinity could lead to a unique solution for TGBMs orthometric heights;
    - RH 2: Proper combination between all available geodetic observations and data per a TG station could lead to a Best Linear Unbiased Estimation (BLUE) solution for the orthometric height of every TG station;

- RH 3: The application of all available geodetic data in TG stations - having different nature, accuracy, resolution and etc. - could lead to a unique, unbiased and improved VD problem solution in the form of very precise and high resolution Geoid Model (GM).

- **THE MAIN RESEARCH HYPOTHESIS** is: 'Is it actually possible and under what conditions, a locally/regionally determined gravimetric geoid (as a 3D vertical datum) to coincide with the corresponding GPS/GNSS/Leveling derived geoid (based on GPS/GNSS observations on an improved version of KSA-VRF14. without any additional transformations?'.

- **THE MAIN RESEARCH CHALLENGE, CONCEPT AND PROBLEM:** For a long time, the only way of definition of VD necessary for height system reference has been restricted to averaging TG records per certain time period for definition of the height above MSL. The application of this approach for more than one TG stations leads to a significant discrepancies and distortions in height determination with regard to orthometric heights determined by a fixed LSA applied for one TG station. Fixing one TG station leads to the concept of 1D Vertical Datum. Those discrepancies are considered to be a result of existing differences in DOT/SST with regard to geoid.

How to combine all available geodetic earth observations and information at the entire KSA-NTGN to determine very accurate geopotential values/numbers & orthometric heights per every TGBMs and how to incorporate this information to the new 3D KSA-VD determination in the form of a new very precise geoid having high resolution, globally referred geoid model and in the new KSA National Vertical Reference Frame (KSA-NVRF)? *The main research problem is very closely tightened to the main research hypothesis, which nowadays is challenging the entire Earth geodetic observation community.*

## 2.2. Research aims and objectives

The following overall, detailed and specific aims and objectives have been defined below.

**OVERALL OBJECTIVE:** *SIGNIFICANT IMPROVEMENT* OF EXISTING KSA-VD & KSA-VRF14 BY DEVELOPMENT OF A CONCEPT AND CORRESPONDING METHODOLOGY FOR COMBINING ALL AVAILABLE GEODETIC EARTH OBSERVATIONS FOR TGBMs GEOPOTENTIAL VALUES/NUMBERS & ORTHOMETRIC HEIGHTS ESTIMATION AND CONSEQUATIVE LEAST SQUIRES ADJUSTMENT (LSA) BY CONSTRAINING GEOPOTENTIAL VALUES/NUMBERS/SHIFTS AT EVERY TGBM.

- The overall objective consists of several detailed objectives as its components and can be defined as:
  a. Improvement of precise leveling observations by introducing more precise leveling corrections based on 1) geographically distributed models for refraction corrections (not just one constant refraction coefficient per

entire KSA) and 2) new airborne gravity data per BMs without observed gravity values;

b. Improvement of Dynamic Ocean Topography/Sea Surface topography per TG station by utilizing TG record data and Satellite Altimetry data analysis together with precise ellipsoidal heights and the most recent Global Geopotential Models (GGMs);
c. Definition of a new multi-Vertical Datum Problem (m-VDP) based on all available types of geodetic Earth observations;
d. Finding numerical solution of the m-VDP and developing a methodology for combining all data and scenarios for height system unification;
e. Conducting a numerical experiment over KSA for computing geo-potential numbers and shifts per every TG station and LSA of KSA National Vertical Network (KSA-NVN) by fixing/constraining the geo-potential numbers or shifts at KSA-NTGN stations;
f. As a secondary objective is considered, providing recommendations how to utilize the developed concept and methodology for future improvement of KSA-VRF14 and in the determination of the new KSA-GEOID20;
g. Providing conclusions regarding the applicability of the developed methodology including new findings for:
    - explaining the existence of historical shift of 0.79 m between Red Sea and Arabian Gulf MSL and between corresponding old KSA Vertical Reference Systems – Jeddah'69 and SVD71/78;
    - estimating the actual difference between the MSL, DOT and SST at Red Sea and the ones at Arabian Gulf based on a complex analysis of all available data at KSA-NTGN;
    - regarding the quality and the relationship between the new solution of m-BVP, the current KSA-VRF14, the old KSA heights systems Jeddah'69&SVD78 and the MSLs at Jeddah and Ras Tanura TG stations;
    - applicability of the new methodology for combining all data and scenarios for height system unification for m-VDP solution.

- Based on the objectives discussed above, the following specific research aims can be formulated, showing how to reach the overall and detailed objectives:
    - Research study about possible theoretical and application improvements of all data available at TG stations according to state of the art achievements per every type of GEOs;
    - Development of processing strategy for combination of all available data and information at TGBMs to estimate their geo-potential values/numbers/shifts and orthometric heights;
    - Incorporation of estimated TG geopotential numbers/orthometric heights into a LSA of KSA-NVN as the most important part of *improved* KSA-VRF14 definition linked to the most suitable global, regional or local (national) geoids;
    - Validation of new m-VDP solution;
    - Recommendations for future research aims linked to the applications of developed methodology:

- To determine a new very precise hybrid KSA-GEOID20 linked to orthometric heights of the improved KSA-VRF14;
- Definition of a new 3D KSA-VD in terms of the best fitted and globally referred KSA-GEOID20;
- Final definition of improved KSA-VRF14 as an important component of SANSRS;
- Transition to a Very Precise Time Varying Geoid (global, regional, national and local) as a KSA Dynamic Vertical Datum (KSA-DVD) changing in time.

## 2.3. Structure of the thesis

This research is structured in an introductory Chapter I (the current chapter), four substantial chapters, and conclusive remarks collected in the final Chapter VI.

- Essential theoretical fragments providing the necessary background of the research are given in Chapter II, where are outlined important aspects of:
  - geodetic coordinate systems;
  - fundamentals of physical theory of the Earth's shape;
  - theory of heights and vertical datums;
  - most commonly used traditional observation techniques and advanced data collection methods.
- Chapter III deals with various geodetic methodologies used to establish feasible approaches for solving the vertical datum problem in a new multilateral way, by using the specific advantages of the data sources available recently:
  - global geopotential models;
  - inland terrestrial (precise levelling, gravity, GNSS) and airborne (gravity) observations;
  - coastal (tide gauges) data;
  - sea-surface (satellite altimetry) observations.
- Chapter IV is focused at some practical aspects, related to the geodetic infrastructure necessary to provide the empirical data for solving the vertical datum problem, in particular in the Kingdom of Saudi Arabia.
- The outcome of all numerical experiment data included in this research is analyzed thoroughly in Chapter V, stressing upon various important details of the implemented observation techniques which contribute for achieving results of better quality and optimize the level of use of the available observation data.
- In the last chapter, the results of this research are summarized and interpreted with regard to the possible implementation for establishing of a new state-of-art vertical datum of the KSA. Some practical recommendations related to realization of the VDP solution strategies proposed in Chapter III are also given. A critical assessment of the achieved fulfilment degree of the research objective and aims, along with author's contribution claims are included as well.

Essential theoretical fragments providing the necessary background of the research are given in this chapter, where are outlined important aspects of:

- geodetic coordinate systems;
- fundamentals of physical theory of the Earth's shape;
- theory of heights and vertical datums;
- most commonly used traditional observation techniques and advanced data collection methods.

Full details about geodetic backgrounds are not presented here but can be found in the full thesis version.

## 2. Major types of vertical datums existing in the past, current time and available in the future

Fig. 2.26 presents a brief summary of vertical datum concept development. The current research study is utilizing *the several TG stations from the past* but in the form of a **m-VDP** by fixing geopotential numbers per every TGBM. But, for geopotential values/numbers determination all available Geodetic Earth Observations (GEOs) are used together with all the major scenarios for height system unification.

Also, in a global scale, the third option of a Global Precise Vertical Datum becomes more and more popular in the form of so-called **I**nternational **H**eight **R**eference **S**ystem (IHRS). The entire gravity field, geoid and height system geodetic community is putting very significant efforts in the definition and establishment of the first realization of IHRS (Sanchez, 2013; Sanchez and Sideris, 2018).

From IHRS point of view, the proposed combined application of all available data and scenarios for the definition of a regional VRF can be considered as:

- a useful contribution to the unification of all regional and national VRFs through the global IHRS definition and the corresponding GGM;
- an easy adaptation of newly improved KSA-VRF14 (by utilizing the proposed in this thesis methodology) to the IHRS just by a simple comparison of the corresponding geopotential values based on a very precise GGM;
- a good example for other regional or national VRFs and VDs definition like future realizations of European EVRS & Geoid, Australian GDA & AusGeoid, Canadian CGVD, NAVD of the USA and etc.

## 3. Conclusion: Past, present and future of vertical datums

In the past, local vertical datums have been established and used, comprised typically of a bench mark linked to a tide gauge where mean sea level is defined using hourly data, collected over several years. Differences between local datums (in different countries) due to Sea Surface Topography (SST) or Dynamic Ocean Topography (DOT) are up to 1.0 – 1.5 m. In some cases, e.g. North American Datum 1983 (NAD83), are used several tide gauges which allows an average mean sea level to be computed. However, the SST/DOT problem remains unsolved in this way.

Nowadays, different surfaces are used as vertical datums:

- Reference ellipsoid is used as ellipsoidal height datum, a pure geometry solution related to GNSS positioning.
- Quasigeoid and ellipsoid are used as normal height datums, respectively for the Earth surface and the telluroid.
- Very precise geoid is used as vertical datum for orthometric and dynamic heights, but SST/DOT still needs to be accurately known.
- A global precise datum (global geoid) based on GOCE data would be unifying local vertical datums defined by tide gauges after subtracting precise SST values from tide gauge data.

In the future further vertical datum enhancements might be expected in terms of:

- Precision and unification - Very precise local and regional geoids referenced to a global vertical datum;
- Temporal definition – Time varying geoid as a dynamic vertical datum, changing in time.

## 3.4 Contribution to the Geoid

The following is considered as a summary of possible applications of TG records analysis to geoid determination. It will be incorporated in this thesis research and presented in the methodology section of Chapter III, in the numerical experiment results analysis and as a part of main thesis contributions, specifically:

- To provide local vertical datum for orthometric heights, necessary for hybrid geoid determination;
- To provide reliable MSL and study the Sea Surface Topography (SST) or Dynamic Ocean Topography (DOT) in order to determine the separation between MSL and a geoid model;
- To investigate time variations of MSL together with GPS and absolute gravity data to determine a dynamic geoid – towards time varying geoid;
- By combining TG data with altimetry data and GOCE data to derive SST (DOT) – to refine the geoid at TG and to introduce new local vertical datum related to the global vertical datum.

## 3.5 Contribution to 1D Vertical Datum definition

TG observations must be related to some fixed local reference point. A Tide Gauge Bench Mark (TGBM) is used as the primary reference point of a Vertical Reference Frame (VRF) - an realization of Height/Vertical Reference System. It has to be clearly marked on a stable surface.

The top surface of a TGBM with assigned height above mean sea level (MSL) is the actual Local Vertical Datum for certain vertical network and height system.

The historical definition of local vertical datum is realized by MSL through simple averaging of hourly TG data during a year or longer time span.
Due to SST (DOT), local vertical datums are not consistent up to 1 m, which leads to up to 1 m difference between the heights of a BM tighten to two different vertical datums.

This can be observed even for islands which are close each other. Many examples exist like US/CANADA NAD28 or existing slope in UK Ordnance Height System.

TGBM is linked by leveling to a GPS site in order to get the local vertical datum and MSL in a global reference system to compare with altimetry, GOCE and GRACE data.

The MSL as a local vertical datum, related to a global reference system, is used for determination of the shift of the local vertical datum with regard to a global vertical datum (very accurate global geoid model). This will lead to unification of local vertical datums.

### 3.6 Contribution to the Geoid determination as a 3D Vertical Datum

Similar to 3.5. the following is considered as a summary of possible applications of TG records analysis to geoid determination. It also will be incorporated in this thesis research and presented in the methodology section of Chapter III, in the numerical experiment results analysis and as a part of main thesis contributions, specifically:

- To provide local vertical datum for orthometric heights, necessary for hybrid geoid determination;
- To provide reliable MSL and study the Sea Surface Topography (SST) or Dynamic Ocean Topography (DOT) in order to determine the separation between MSL and a geoid model;
- To investigate time variations of MSL together with GPS and absolute gravity data to determine a Dynamic Geoid – towards time varying geoid;
- By combining TG data with altimetry data and GOCE data to derive SST (DOT) – to refine the geoid at TG and to introduce new local vertical datum related to the global vertical datum.

## Chapter III: Applied methodologies

The following general methods, approaches and methodologies have been utilized during the thesis research:

- *Boundary Value Problem (BVP) Approach* – consisting of BVPs – mathematical formulation; Geodetic Boundary Value Problem (GBVP); Stokes' GBVP; BVP solutions as Solution #1: Spherical Harmonic Series Expansion and Solution #2: Stokes' integral using the Stokes' function;
- *Global geopotential modelling (GGM)* using spherical harmonic functions – in terms of Earth's gravity potential expansion in spherical harmonic series; Spherical functions basics and Legendre polynomials; 3D Mathematical Harmonic representation for a gravitational potential of non-rotating sphere; Zonal, sectorial and tesseral harmonics; Representation of Earth's geopotential and geoid as solid spherical harmonic functions linked to geoid wavelength structure; Theory and examples of GGMs; Satellite gravity missions CHAMP, GRACE, GOCE with space-born sensors and spectral content of contributions to GGMs;

- *Refraction coefficient determination and modelling* - applied for the territory of the Kingdom of Saudi Arabia; Refraction coefficient modelling;
- *Tide Gauge Wavelet Multi-resolution Analysis* (WMRA) for Mean Sea Level Determination and Vertical Datum Definition of the Kingdom of Saudi Arabia (KSA) – including WMRA for filtering out timely correlated errors in TG and SA data; Application of WMRA for combined tide gauge and Satellite Altimetry data analysis; Methodology for jumps and steps detection in TG signal; Utilization of WMRA of TG &SA data for *GNSS like approach for SST determination* at TGBM;
- *KSA-VRF definition and Height System Unification* (HSU) *methodology* - utilizing satellite altimetry, GOCE data and in-situ leveling and gravity observations; Ellipsoidal height estimation from satellite altimetry; HSU over the KSA with GOCE, TG, altimetry in-situ gravity and levelling data; HSU theory and Wo LVD estimation; Approach for combination of differential levelling, geopotential values and surface gravity derived from EGM2008 and GOCE/EGM2008 using a least-squires adjustment.

In the rest of this chapter only new specific methodologies applied in the thesis will be described in details.

### *D.Tide Gauge Wavelet Multi-resolution Analysis for Mean Sea Level Determination and Vertical Datum Definition of the Kingdom of Saudi Arabia*

Today, satellite altimetry (SA) over oceans and seas is being considered as complementary to the space positioning techniques over land (SLR, GPS, DORIS, and VLBI) which, in combination with tide gauges (TG), can be employed in vertical crustal motion, mean sea level rate and vertical datum definition studies (see, e.g., Cazenave et al., 1999; Nerem and Mitchum, 2002; Rangelova et al., 2006). While altimetry determines sea level in a geocentric coordinate system, tide gauges are attached to the deformable solid Earth and measure the sea level with respect to it. Differences of the rate of change of the two signals at the tide gauge location determine the absolute uplift (or subsidence) of the Earth's crust. For this purpose, the altimeter instrument drift should be precisely known and the periodic variability in the altimetric and tide gauge sea level signals be either removed or cancelled out. Differences between altimetric sea surface heights and tide gauge sea levels could be analyzed for the purpose vertical datum monitoring as well.

In semi-enclosed sea regions such as the Red Sea and Arabic Gulf, long-term oscillations in the altimetry signal cannot be averaged out when linear trends are sought in contrast to global studies of sea level rise. The presence of non-linear trends in the sea level signal, may obscure the estimation of the rate of change of the sea level using least-squares regression. Moreover, the absence of accurate long term tidal corrections for Red Sea and Arabic Gulf can be a critical factor for the estimation of trends from short time span altimetry data since short-periodic oscillations may not average out. If differences between altimetric and tide gauge sea level signals are utilized, some of the periodic components may cancel out. However, to apply this method, only the last 3 decades of SA data can be used. For the current TG data analysis, we are utilizing only 7 years of Red Sea and Arab Gulf data for vertical datum definition.

The aim of this study is to investigate the capabilities of the wavelet-based multiresolution analysis to identify and remove most of jumps, steps and residual periodic components from the sea level signal prior to estimating the sea level trend. Since wavelets have good localizing properties both in the time and scale domains, residual periodic components present in the signal can be detected. This is achieved by imposing restrictions, i.e., threshold values, on the detailed coefficients at different levels of decomposition. Examples of wavelet multi-resolution analysis for analysis of GGMs can be found in Peidou et al. (2015 a, b).

### *E. KSA-VRF definition and Height System Unification with satellite altimetry, GOCE data and in-situ leveling and gravity observations*

This methodology has been developed by the GDG technical advisory team (Vergos at al., 2016), (Vergos at al., 2017). It is presented here only for completeness of the thesis and to support the emphasis of other chapters and contributions focusing on main contribution – definition of new type of vertical datum and finding solutions for it.
It summarizes the methodological steps for the KSA-VRF definition based on satellite altimetry and GOCE data utilizing also in-situ observations from TG's, leveled differences and gravity. The analysis of satellite altimetry data is given, based on both time- and scale- decomposition in order to derive the ellipsoidal height at the TG locations at various epochs from mono- and multi-mission altimetry. The focus is based on the entire set of satellite altimetry data starting with GEOSAT in 1985 up to today with the latest missions of Jason-2, Cryosat2 and Saral/AltiKa. An overview of height system unification given the available leveled differences at the TG locations is provided with some first comments and proposals for the determination of the zero level geopotential value $\widehat{W}_o^{LVD}$ for the Saudi Arabia VRF in the form of geopotential differences $\delta W_o$ relative to the existing leveling network and a global height system in the form of a WHS.

## 2.1. Height system unification over the KSA with GOCE, TG, altimetry in-situ gravity and levelling data

The use of heights is of main importance for a wide range of geodetic, surveying and engineering applications. In the case of orthometric heights, i.e. heights above the geoid, differences are determined nationwide by conventional spirit leveling accompanied by gravity measurements along dedicated traverses. The orthometric heights of all established benchmarks are then obtained, through a LS adjustment of the entire vertical network, as height differences with regard to a selected BM that serves as the origin point of the country's vertical reference system. It has been customary for the origin point to coincide with a or several TG station(s) at which the local MSL has been determined over a long period of time – the latter provides the primary realization of a zero-height reference surface from which all orthometric heights are referred and measured thereafter. In this way, the orthometric height of a point $P$ on the Earth's

surface with regard to an LVD is obtained, in principle, through the geopotential difference between the Earth's gravity potential $W_P$ at that point and the reference geopotential value $W_o^{LVD}$ on the associated zero-height surface of the LVD divided by the mean gravity along the corresponding plumbline segment (Heiskanen and Moritz 1967, sect. 4-4).
For the pre-definition and realization of the KSA-VRF a zero-height geopotential value needs to be associated with a TG BM and the respective MSL at this station or a number of TGs can be used. In essence, at the present time that the KSA-VRF has not been tight yet to a TG MSL or to that of several TGs each station/BM has its own LVD, which is not connected to the rest and to the origin of the KSA LVD at Jeddah. Fig. 3.22 and Fig. 3.23 below summarize the situation over KSA, where each LVD is realized by the local orthometric heights (leveled differences) at the respective leveling BMs.

In order to achieve the unification of the KSA LVD, one needs a high-accuracy geoid, so that it will serve as the reference with regard to the local offsets will be determined. This is the place where GOCE data in the form of GGMs will be used, mainly coming from the latest R5 GGMs based on the TIM, DIR, GOCO and SPW development approaches. An example of GGMs applications can be found in Tziavos et al. (2015).

The KSA-VRF will follow two possible approaches for the estimation of the KSA LVD zero-level geopotential value $W_o^{LVD}$ and the estimation of the geopotential differences at each TG $\delta W_{TG_i}^{LVD}$, using GOCE-based GGMs, collocated ellipsoidal heights at the TGs, leveled difference, gravity data, EGM2008 and the latest KSA gravitmetric geoid Local Gravimetric Geoid.

The first approach consists of an estimator based on an LS adjustment of the leveled height differences at the TG location and (i) the EGM2008 GGM, (ii) a hybrid GOCE/EGM2008 GGM used to derive the gravity potential $W_i$ at each TG BM. In this approach there is no need to use geoid heights in estimating the zero-level geopotential, so the inherent uncertainty for the topographic effects on geoid heights when evaluating them from a GGM is avoided. The second approach is based on the differences between geoid heights from (i) the EGM2008 GGM, (ii) a hybrid GOCE/EGM2008 GGM, (iii) Local Gravimetric Geoid, the satellite altimetry derived TG ellipsoidal height, the TG orthometric height and either local gravity data or synthetic gravity observations. The estimate of the mean offset can then give a direct relationship between the local LVDs at each TG and their connection to a unified certain $W_o$ value.

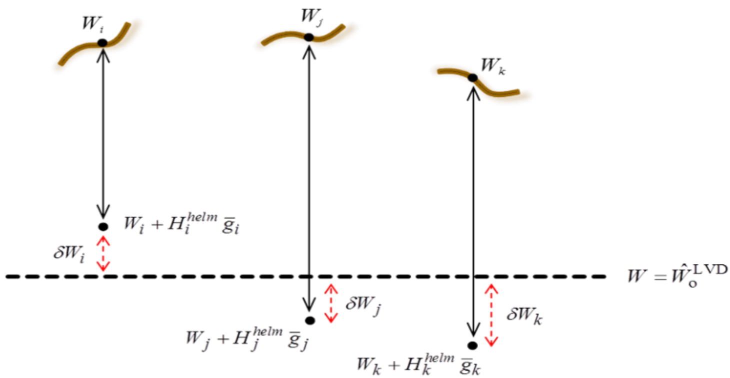

Figure 3.22. Zero-height geopotential value determination from a network of stations.

## 2.2. Theoretical background for HSU unification and $\widehat{W}_o^{LVD}$ estimation

Let us assume that physical orthometric heights $H_i$ ($H_i$ is the $H_{MSL}^{i}$ of the respective 12 TG stations over the KSA) are available over a network of benchmarks (BMs) $i = \{1,2,3,\ldots,m\}$, derived by traditional spirit leveling, with their orthometric heights referred to the mean sea level realized by tide-gauge stations. Some theoretical and application aspects can be found in Vergos et al. (2014, 2015) The latter forms the origin of the LVD in the region under study, which all orthometric heights are referred to, with a generally unknown, zero-level geopotential value $W_o^{LVD}$. An estimate of $W_o^{LVD}$ can be achieved, following two approaches, when for the same BMs ellipsoidal heights $h_i$ (being the altimetry derived ellipsoidal height $h_{MSS}^{alt} = H_{MSL}^{TG} + \zeta^c$ at each TG) derived by satellite altimetry, surface gravity $g_i$ and the geopotential $W_i$ computed from a GGM, are available.

## 3 Concluding remarks

For repeating all computations in all scenarios and sub-scenarios in this Chapter the substitution of EGM2008 by any other model will provide corresponding geopotential values, numbers and shifts with regard to the GGM applied. Those approaches have been applied in Chapter VC for XGM2016 and EIGEN 5c GGMs.

The ellipsoidal height of MSl and TGBM and the DOT/SST derived with this methodology will be utilized in next Chapter_IIIF and Chapter V for finding solutions for multi-vertical datum problem. The implementation of SA/TG derived ellipsoidal height of TGBM into determination of geopotential shift of local vertical datum from regional or global vertical references or Geoids can be considered as contribution to m-VDP solution methodology, which is presented in Chapter IIIF.

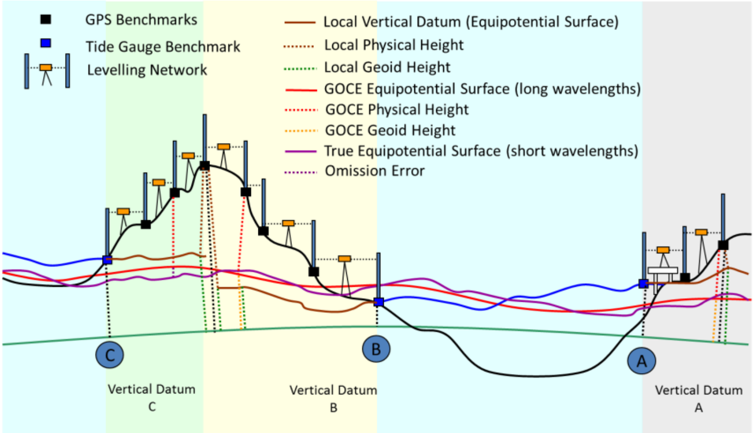


Figure 3.23. LVDs and GOCE-based geoid aid to LVD unification (Gruber et al. 2014).

## *F. New multi-Vertical Datum Problem definition and proposed solution*

### 1. Multiple-Vertical Datum Problem

#### 1.1. Problem statement

Having many different types of collocated geodetic observations at a TG station would allow the computation of the orthometric height of a BM from the TG Network to be determined up to the highest possible accuracy requirements.

Main research task is to find a solution of the problem how to combine all existing and near future available geodetic earth observations and information at the entire KSA-Tide Gauge Network (KSA-TGN) to determine very accurate geo-potential numbers/orthometric heights for GCS TG stations and how to incorporate those heights to the new KSA Vertica Datum determination in the form of a new very precise, high resolution, KSA globally referred geoid model and in the new KSA National Vertical Reference Frame (KSA-VRF).

Main research question is how to combine different geodetic earth observations for definition of a Vertical Reference Frame based on the approach for unification of Local Vertical Datums (LVDs)

- The unification of LVDs usually is designed for merging already existing local/regional vertical reference systems and frames;
- According to Amjadiparvar et al. (2016) there are three main strategies for LVD unification according to:
  - First strategy: a direct connection by levelling and gravimetry. At a regional or continental level, two or more LVDs can be connected on land by precise levelling in combination with gravity measurements complemented with GPS/GNSS determined ellipsoidal heights;

- Second strategy: the oceanographic approach. LVDs without connected points on land can be connected by means of oceanic levelling based on ellipsoidal heights from GPS/GNSS observations or SA and TG records analysis;
- Third strategy: the geodetic boundary value problem (GBVP) approach. It connects the datums by solving the GBVP and computing LVD offsets with respect to a global equipotential surface.

- Main characteristics of the newly defined problem:
  - Unification of several LVDs will be applied for one large regional VRF definition based on TG Network spanned on two water basins – Red Sea and Arabian Gulf;
  - All three basic strategies usually applied separately will be applied together, taking advantage of the fact that Kingdom of Saudi Arabia (KSA) has established in a relatively short time all necessary geodetic networks with high accuracy requirements; Also, KSA has collected (or very soon to be collected) all data and information (mentioned above) necessary for simultaneous application of all described above strategies;
  - Usually, unification is applied on:
    1) several LVDs and each one is fixed to 1D Vertical datum at specific TG station,
    2) *BUT in the new problem the question is how to produce* ***ONE Regional Multiple LVDs (m-VD)*** *based on several TG stations linked through many different types of data and information*.

- Global Vertical Datum Problem: A 3D vertical datum is an equipotential gravity field surface with assigned a constant geopotential value $W_0$, which serves as the reference surface for physical heights (see Sanchez and Sideris, 2018).
  - Traditionally, a local vertical datum was based on the equipotential surface of the gravity field of the Earth that coincides with an *idealized mean no-motion* surface of the oceans;
  - In such case, $W_0^J$ is the geo-potential of the mean sea level at the origin of a single levelling network noted as J – or many different local, national and regional LVDs exist,
  - BUT A single unified and global vertical datum is required to provide a reference surface for monitoring the solid and fluid Earth and enabling the connection between the geometric and gravity reference frames;
  - Regionally, unified datums are required for international engineering projects, flooding control, coastal hazard studies, unification of national gravity anomaly databases, and improvement of continental geoid models

- The problem of the unification of LVDs is referred as a **V**ertical (or **H**eight) **D**atum (VD/HD) problem.

**NEW REGIONAL MULTI-VERTICAL DATUM PROBLEM:** To define a m-VRD based on the geopotential offsets $\delta W_0^N$ at $N$ TG stations with regard to a global geopotential value $W_0$, determined by all types of available geodetic observations and computation strategies.

## 1.2. Mathematical Representation of m-VDP

The following general mathematical formulation in (3.47) is possible for a m-VDP per *TG<sub></sub>*$TG_i$:

$$
(3.47.)\quad \left|\begin{array}{c}
\Delta T = 0, Laplace's\ equation \\
\Delta g = -\frac{\partial T}{\partial r} - \frac{2}{r}T, gravity\ anomaly\ as\ boundary\ condition \\
r = R\ is\ Earth's\ \ mean\ radius\ of\ a\ sphere\ \sigma \\
T(r)_{r\to\infty} \to 0, assimptotic\ condition \\
To\ find\ T\ outside\ and\ \ on\ sphere \\
\boldsymbol{Additional\ pointwise\ boundary\ conditions\ at\ TG\ i}: \\
h_{GNSS} = H^{Helmert} + N_{Local} \\
h_{TG\&SA} = H^{Helmert} + N_{Local} \\
N_{Local} = \frac{T}{\gamma_0} + \frac{\delta W_0^{LVD}}{\gamma_0} \\
g = -\frac{T}{H^{Helmert}} - 0.0424H^{Helmert} + \gamma
\end{array}\right.
$$

It is visible that m-VDP is an overdetermined BVP which can be solved either with least-squires adjustment in one step or by different separate strategies and a combination of them. *When only SA and TG data are available the case has been discussed in Chapter_IIID.* For finding 3 separate solutions the procedure developed in Amjadiparvar et al. (2016), Hayden et al. (2012), from Michael Sideris group at University of Calgary will be utilized. In this thesis, the combination of separate solutions will be applied and the application of LSA in one step will be recommended after all data from airborne gravity project became available. Those data are necessary for efficient application of Stokes' integral in vicinity of every TG station by utilizing dense airborne or land gravity data for solutions of Stokes' BVP.

## 1.3. Solutions for m-VDP

Three strategies will be considered in finding solutions for m-VD following the strategies developed in Amjadiparvar et al. (2016).

### 1.3.1. Strategy 1: A direct connection by levelling, gravimetry and GNSS/Leveling data - determination of $W_0$ shift of GNSS/Leveling geoid with regard to global geopotential model

**Delta $W_P$:** The core formula applied according to Amjadiparvar et al. (2013a) reads:

(3.48.) $$\delta W_P = \gamma \delta N_P = -W_0 + U_0 - \gamma(h_P - H_P - N_P),$$

where:

(3.49) $$N_P = N_P^{GGM} + \frac{\delta GM}{R\gamma} - \frac{\Delta W_0}{\gamma}$$

is global geoid value with $N_P^{GGM}$ generated from a GGM using height anomaly at point P by formula

(3.50.) $$N_P^{GGM} = \zeta(\varphi_P, \lambda_P, r_P) + \frac{\delta g_B}{\gamma_m} H_P ,$$

$\delta g_B$ – Bouguer correction, $\gamma_m$ – normal gravity at the middle between telluroid and ellipsoid; $\delta GM$ is the difference between gravitational constant of Earth and those of the reference ellipsoid used (GRS1980); $R, \gamma$ are the mean radius of the Earth and normal gravity at the ellipsoid at P; $\Delta W_0$ is the difference between the global geoid geopotential $W_0$ (62636856.00$m^2/s^2$) and the ellipsoid normal geopotential $U_0$. Other parameters are: $h_P$ - the ellipsoidal height at point P; $H_P$ - orthometric height in KSA-VRF14 at point P; $\delta N_P$ - weighted (by cosine of latitude) mean datum offset (the offset of GPS/Leveling geoid above the GGM) at point P:

(3.51.) $$\delta W = \gamma \delta N = \gamma \frac{\sum_{P=1}^{n} \{\frac{\delta W_P}{\gamma}\} \cos(\varphi_P)}{\sum_{P=1}^{n} \cos(\varphi_P)}.$$

### 1.3.2. Strategy 2: The oceanographic approach - LVDs without connected points on land by means of oceanic levelling

***$DW_P$*:** The core formula applied according to Hayden et al. (2012) reads:

(3.52.) $$W_P = W_N - \gamma(SST_P),$$

where: $SST_P$ is the Sea Surface Topography at P at water surface from global model or TG records; $W_N$ - geopotential of an equipotential surface trough the MSL at Jeddah TG, the local vertical datum (LVD) for KSA-GEOID17, with:

(3.53.) $$ST_P = h - N - CD + Z_0,$$

$h$ - ellipsoidal height at BM $P$, $N$ - geoid height at BM $P$, $Z_0$ - height of MSL above the local chart datum, $CD$ - orthometric height in local chart datum at BM $P$, defined as follows:

- if chart datum is TGBM, $CD = 0$ and $Z_0 = -R$ (from TG analysis);
- if chart datum is MSL in Jeddah, $CD = H$ (in VRF 14) and $Z_0 = MSL$ (MSL – mean sea level at Jeddah TG, satellite altimetry analysis);

$R$ - TG reading with regard to TGBM at reference epoch after trend estimation of wavelet filtered TG signal

(3.54.) $$W = \frac{\sum_{P=1}^{n} W_P \cos(\varphi_P)}{\sum_{P=1}^{n} \cos(\varphi_P)}.$$

**STRATEGY 2B:** Tide gauge record and satellite altimetry analysis by using WMRA for mean sea level and its trend determination

**Methodology description for WMRA procedure**: For detecting jumps, steps and as smoothing filter has been developed and presented below. For the numerical experiment Daubechies 10 wavelets have been used and corresponding scaling and wavelet coefficients are determined in the following steps:

### 1.3.3. STRATEGY 3: The geodetic boundary value problem approach connecting the datums by solving the GBVP and computing LVD offsets with respect to a global equipotential surface – Short Description

The GBVP approach as a rigorous method for computing the relationships between separate vertical datums in terms of offsets to a common global equipotential surface defined by a GOCE based GGM geoid (Amjadiparvar et al. 2013, 2016).

- This solution takes into account: Indirect bias term; GOCE geoid omission error – effect of spherical harmonics above $N_{max}$ – the maximum degree and order of GGM; Effect of systematic errors and distortions from leveling datum (GNSS/leveling); Effect of geodetic errors on the datum – LVD offsets.
- GBVP approach recommended to be applied and investigated over KSA with new air-borne gravity anomalies (after they become) available

**STRATEGY 3B:** The GBVP approach connecting the datums by solving the GBVP and computing LVD offsets with respect to a global equipotential surface (GGM) – Theoretical Background

**Gravimetric Geoid $N_P$:** The core formula applied according to Amjadiparvar et al. (2016):

(3.55.) $$N_P = N_P^{GGM} + \left[\frac{\delta GM}{R\gamma} - \frac{\Delta W_0}{\gamma}\right] + \delta N^j + \delta N_{P,res}^j + \delta N_P^{ind},$$

where: $N_P$ - gravimetric geoid at point $P$; $N_0 = \frac{\delta GM}{R\gamma} - \frac{\Delta W_0}{\gamma}$ - first degree term; $N_P^{GGM}$ is generated from a GGM geoid height at point $P$; $\delta N_P{}^{ind}$ - indirect bias term (can be neglected):

(3.56.) $$\delta N_P{}^{ind} = \sum_{i=1}^{J} S_P^i \delta N^i,$$

with $S_P^i$ - integral of Stokes only function over area $i$; GOCE geoid omission error, $\delta N_{P,res}^j$, are residuals due to the modified kernel which does not contain harmonic degrees less than GGM $N_{max}$.

The effect of systematic errors and distortions from leveling datum (GNSS/leveling offset of gravimetric geoid) is

(3.57.) $$h_P - H_P^j - N_P^{GGM} - N_{P,res}^j - N_0 = (1 + S_P^j)\delta N^j + S_P^i \delta N^i \,.$$

The effect of geodetic errors on the datum is LVD offset for $j$-th datum zone:

(3.58.) $$\delta N^j = \frac{\delta W_0^j}{\gamma} \,.$$

**STRATEGY 3C:** The GBVP approach connecting the datums by solving the GBVP and LSA with fixed LVD offsets per every TG with regard to a global equipotential surface (GGM) – *Expected results after air-borne project completed*

- GBVP for KSA TG Network (KSA-NTGN)
  - consists of 12 *LVD* zones *i*: *i = J,Q,J,Y,W,D,M,A,O,T, Mr, K;*
  - spans *50*-100 km around every TG station.

- Least square adjustment with configuration matrix ***A*** according to Amjadiparvar et al. 2016, p.50 and the following ob**servatio**n model:

(3.59.) $$\mathrm{l} = \mathbf{A}\delta \mathrm{N}^{\mathrm{i}}, \quad \mathrm{Q_{ll}},$$

where $Q_{ll} = Q_{hh} + Q_{HH} + Q_{NN}$ is the covariance model with or without indirect term;

- GBVP approach unifies other two strategies:
  - *h* from GNSS - Strategy 1;
  - *h* from TG or SA – Strategy 2;
  - $\delta N_G$ from GGM - offsets with regard to Global Datum;

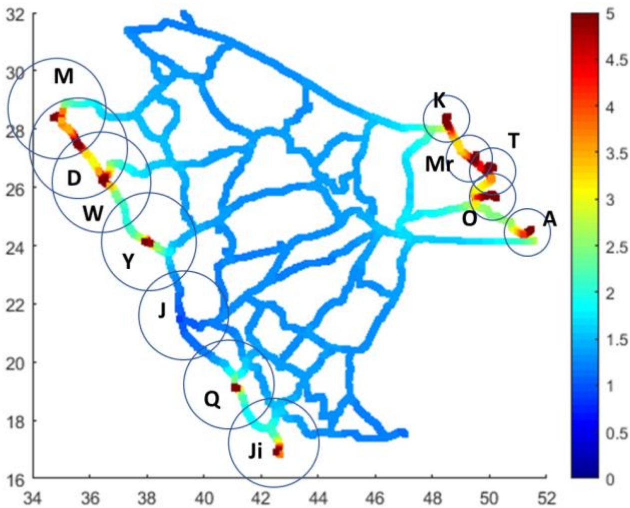


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Figure 3.24. LSA with fixed geopotential numbers shifts per TG benchmarks determined by GBVP in vicinity of every TG station (50 – 100 km radius)

### *1.3.4. ADDITIONAL STRATEGY 4 (RECCOMENDATION): FUTURE APPLICATION OF COMBINING ALL AVAILABLE DATA AND SCENARIOS FOR IMPROVEMENT OF KSA-VRF & 3D KSA-VD (KSA-GEOID)*

The following recommendation for improvement of KSA-VRF could be considered as an important contribution to completing the *entire concept* for combination of different types of Geodetic Earth Observations for definition of a new Vertical Reference System of the Kingdom of Saudi Arabia (KSA-VRS).

KSA-VRS itself is the second component of Saudi Arabia National Spatial Reference System (SANSRS), which is defined as a *core foundation* of the National Spatial Data Infrastructure (NSDI) (see Chapter IV) of the Kingdom according to the VISION 2030.

***RECCOMENTATION:*** *The LSA scheme described in STRATEGY 3C can be applied for combination of all data and strategies utilizing GBVP, GNSS/Leveling and Oceanographic approaches. This a recommendation for a final complete solution of an improved KSA-VRF after:*

- ❑ *finalizing the gravity anomalies from air-born project for estimation of orthometric corrections for BM without gravimetric observations;*
- ❑ *taking into account the new refraction models in the final LSA of KSA-NVN (utilizing Chapter IIIC & Chapter VA).*
- ❑ *final selection of the best performing both in coastal and in-land area over the KSA, up to date GGM [utilizing Chapter III-B,E];*
- ❑ *finalizing the gravity anomalies from air-born project for estimation of orthometric corrections and the $\delta N^{i}_{L}$ local geoid shift per $i^{th}$TG (utilizing Chapter III-A,F);*
- ❑ *finalizing the common analysis of TG and SA data based on up to date available data and taking into account the results of ~ 6 years TG deformation network for tectonic and local vertical movements of TG stations and new estimations of MSL rate (utilizing Chapter III-D,E).*

# Chapter IV: Data and information available for numerical experiment

### *A. National Geodetic Infrastructure (NGI) of Saudi Arabia – source for all data and information*

All different types of data and information exist in the NGI of Saudi Arabia. Having different types of collocated geodetic observations at a TG station would allow the computation of the orthometric height of a BM from the TG Network to be determined up to the highest possible accuracy requirements. One of the secondary application task of this thesis is to utilize the developed methodology for combination of all existing data and information to improve the current vertical reference frame of Saudi Arabia namely KSA-VRF14.

The description below shows general categories of data and information existing in SANSRS and KSA-NGNs

- Dynamic Ocean Topography/Sea Surface Topography values for TGBM-S from the most recent models;
- $\delta N_L$ from Local Geoid - offsets from Local Datum.

The following types of data and information are available at Kingdom of Saudi Arabia Tide Gauge Network (KSA-TGN):

- TG records for more than 7 years (up to January, 2019) at12 TG stations along Read Sea and Arabian Gulf coastal line;
- Precise levelling data for monitoring stability of every TG Network (3.5 years up to now plus 6 months periodic observations till 2020;
- Helmert Orthometric heights above MSL at KSA-NTG in KSA-VRF14;
- Precise gravity values for main TGBMs for every TG station;

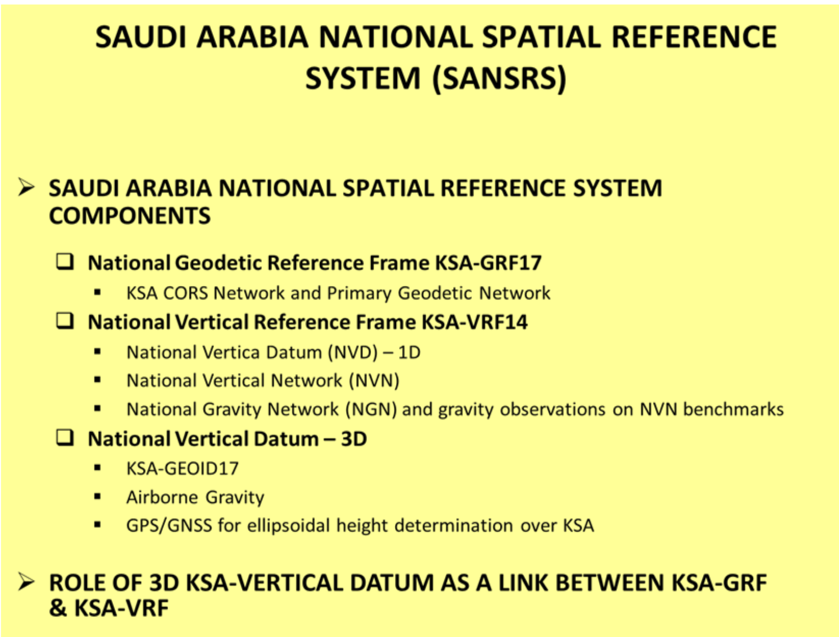



Figure 4.1. General categories of data and information in SANSRS&KSA-NGNs

- Geoid Values at TGBMs from current gravimetric Geoid GRA-GEOID15 and the hybrid geoid KSA-GEOID17;
- Global Geopotential Models (GGM) geoid values for most recent Earth GGMs for TGBM-S for every TG station;

### 1. Saudi Arabia National Spatial Reference System and National Geodetic Networks (KSA-NGNs)

The National Geodetic Infrastructure (NGI) (Al-Kherayef and Grebenitcharsky, 2019) of Kingdom of Saudi Arabia is considered as an important part of the National Spatial Data Infrastructure (NSDI) of the Kingdom providing a *unified, highly accurate and with national coverage* reference for geo-referencing of all data and information in NSDI. The following diagram explains the main components of National Geodetic Infrastructure of the Kingdom of Saudi Arabia - THE SAUDI ARABIA NATIONAL SPATIAL REFERENCE SYSTEM (SANSRS) and THE NATIONAL GEODETIC NETWORKS (KSA-NGNS).

A more detail description about a NGI is provided below according to a *Report in Brief of The National Academy of Sciences© (2010)* describing a precise geodetic infrastructure as:

The geodetic infrastructure provides accurate information about fundamental properties of the Earth as they change over time and has led to many scientific, civil, military, and commercial applications. Numerous agencies and organizations have made valuable contributions to the geodetic infrastructure over the past half century; however, this critical infrastructure is now degrading from age and lack of support. Renewed investment in the geodetic infrastructure is needed to maintain and modernize existing systems and to enable the development of sophisticated new applications with significant economic, national security, and scientific benefits.

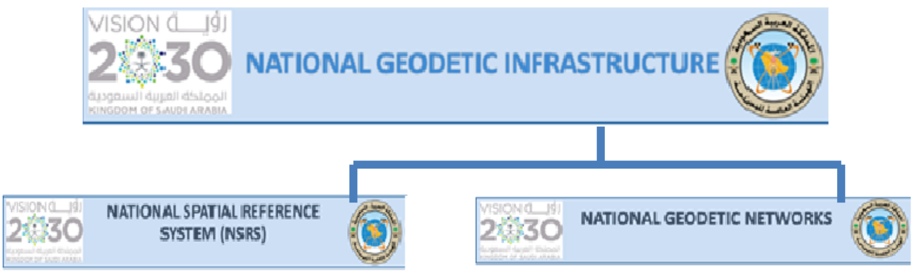



Figure 4.2. National Geodetic Infrastructure of the Kingdom of Saudi Arabia

*NGI is a national geodetic infrastructure because it is designed and established to meet precise geodetic requirements for all type of activities and applications over the KSA by providing the best accuracy and national coverage over the territory of the Kingdom of Saudi Arabia.*

Below, its two main components are briefly presented as:

- combination of Spatial Reference System and Networks which contribute in establishing and supporting positioning applications in horizontal and vertical directions with high quality & accuracy and national coverage;
- it is the foundation of National Spatial Data Infrastructure (NSDI).The National Spatial Reference System has been briefly presented below with its 3 geodetic components.

*SANSRS is a national spatial reference system because it is a **nationally unified**, designed and established to meet precise geodetic requirements for **geo-referencing** for all types of geomatics and geodetic activities and applications over the KSA by providing the best accuracy and national coverage over the territory of the entire Kingdom of Saudi Arabia.*

The National Geodetic Network has been briefly presented below with its 5 geodetic networks:

*Like in the case of KSA-NGI & SANSRS, KSA-NGN is national geodetic network because it is a **nationally unified**, designed and established to meet precise geodetic requirements for all types of geomatics, remote sensing land survey and etc. geodetic activities and applications over the KSA by providing the best accuracy and national coverage over the territory of the entire Kingdom of Saudi Arabia.*

The entire KSA-NGI has been designed and established following the Concept for combining and complementing each other of all possible Geodetic Earth Observations in order to establish a state of the art SANSRS with its 3 major components:

- KSA Geodetic Reference Frame (KSA-GRF);
- KSA Vertical Reference Frame (KSA-VRF);
- KSA Vertical Datum (KSA-VD) and its 3D realization as KSA-GEOID.

It is worth to mention that the author of the thesis had provided a significant contribution to all steps and phases of GNI establishment from the very beginning for: 1) research and development of the concept for combination of all existing and available types of geodetic observations, data and information; 2) strategic planning and every day management of all activities related to GNI establishment as a whole; 3) following up and quality controlling of all GNI related projects completed and still under execution.

The experience gained during last 10 years allowed the author to strongly contribute to the establishment and the definition of KSA-VRF and KSA-VD and motivated him to include them in the formulation of one of the main scientific tasks of the dissertation:

Development of the Concept and the Methodology for: 'Geodetic Earth Observations Combination for Vertical Reference System Definition and its Realization as a Case Study for the Kingdom of Saudi Arabia Vertical Datum and Vertical Reference Frame'.

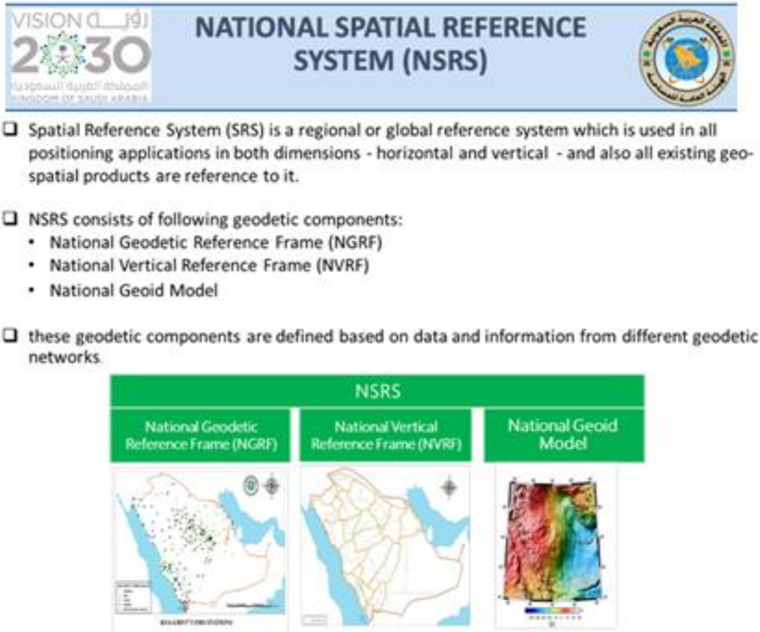



Figure 4.3. Saudi Arabia National Spatial Reference System (**SANSRS**)

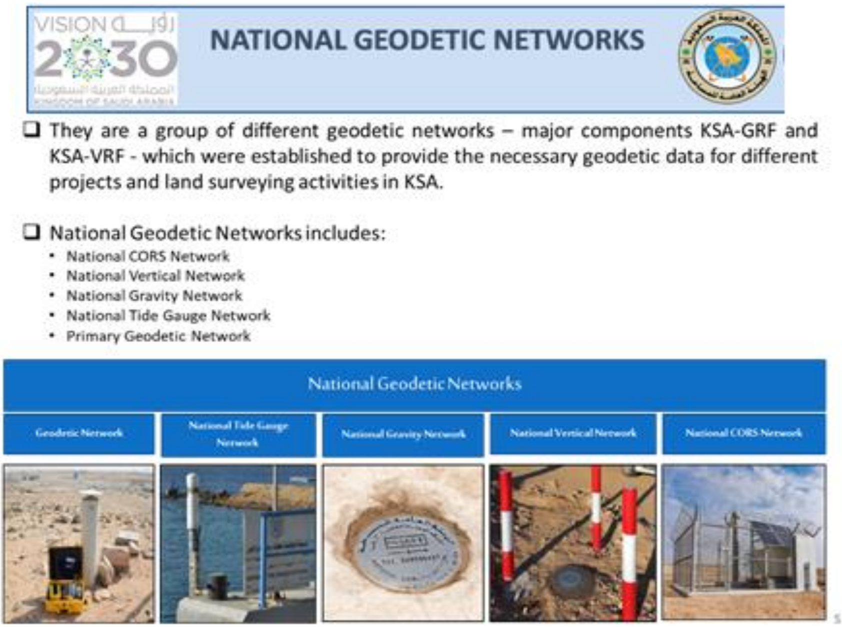



Figure 4.4. Kingdom of Saudi Arabia National Geodetic Network (KSA-NGN)

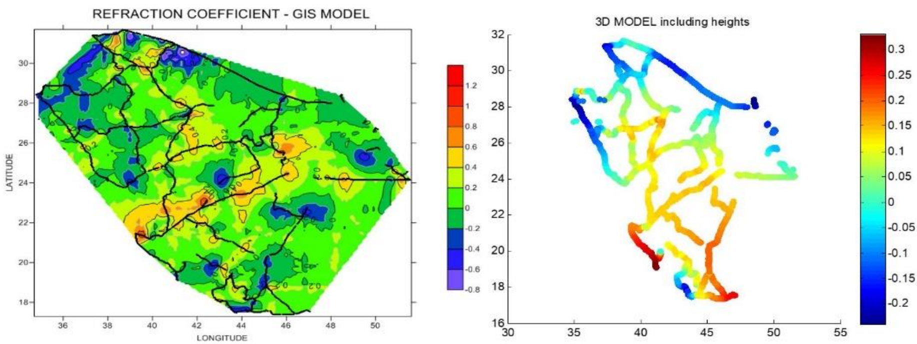
### *A. Refraction coefficient determination*

### 1.3.1. 3D GIS models of refraction coefficient residuals

Two 3D refraction coefficient *residual* models (*with regard to* $C_{weighted\ average} = -0.420$) were computed as:

(5.1.) $$\Delta C_{sec.residual} = C_{section\ moving\ average} - C_{total\ weighted\ average}$$

The first one (a) is location dependent, whereas the second (b) considers also the section's heights and in this way takes into account the second order effect of correlation between the longitude and the height and between the latitude and the height. In addition, the second model gives the best results in terms of validation (see next section) by misclosures along the levelling lines and loops. Both models can be used for precise computations and future research. The underlying shape files could be incorporated in any GIS environment and subjected to further analysis. A corresponding grid file, on the other hand, is more suitable for interpolation of observed values of refraction coefficient.




a) 3D model based on location b) 3D model based on location and heights

Figure 5.1 GIS models of refraction coefficient

## 2. Calculation of refraction corrections and validation of the best model

### 2.2. Validation of different RC models and conclusions for their applicability

In order to validate the best solution for refraction coefficient model, the corresponding refraction corrections per setup were computed using the average refraction coefficient for corresponding levelling section. Computing the misclosures between forward and backward directions per levelling lines together with loop misclosures and the algebraic sum of misclosures along the loops gives an idea about the performance of the corresponding model.

After analysing misclosures along all available levelling lines, the following conclusions were made:

- The refraction corrections per setup are very sensitive to the temperature behaviour. It is recommended that average refraction coefficient and average refraction correction per section to be computed and used in case the measured temperature does not comply with its analytical logarithmic behaviour;
- The greater improvement (60% - 70%) in levelling line misclosures is obtained within the 3D refraction model dependent on height.
- The best improvement is for equivalent height refraction coefficient model reaching up to 70% per observed and 68% per Contractor's values of refraction corrections;
- The new model of refraction corrections for both regular and equivalent height provides greater improvement comparing to Contractor's misclosures. Therefore, this model should be recommended for precise levelling observations, expecting up to 60% decrease in levelling line misclosures.
  Further investigation on the expected improvement by the new refraction model in terms of refraction coefficient effect on levelling misclosures was performed using 27 available levelling loops. Both the algebraic sums of line (forward and backward directions) and loop misclosures have been computed. Results show that the algebraic sum of misclosures (per kilometre) have an improvement of 70%. This corresponds to about 3-4 cm decrease in the misclosures' magnitude. Similarly, the improvement in loop misclosures is 52% or 4-5 mm less than the values obtained by the Contractors. The latter could be explained by existing problems with the temperature measurements, which had led to increase of the refraction correction mean value for levelling section. Similar computations were performed taking into account the effect of the equivalent height. The results, however, show less improvement: 30% and 41%, respectively.
- ***Final conclusions:***

  Based on the study above, the following conclusions could be drawn:

  ○ *An average value of refraction coefficient **C = -0.420** has been determined and can be successfully applied over entire KSA territory for applications requiring less height accuracy;*

  ○ *In case height information is not available, a location-only-based 3D GIS-like model of refraction coefficients residuals model can be applied for reducing of precise levelling observations;*

  ○ *A final 3D residual (with regard to the average C value above) refraction model (including a height correlation parameter) should be considered for computation of refraction coefficients in future very precise levelling works in KSA. The latter contributed to a significant improvement in reducing the levelling section differences and loop misclosures.*

## *B. Improvement of Vertical Datum determination by combination of Tide Gauge and Satellite Altimetry data*

### 2. Dynamic Ocean Topography /Sea Surface Topography determination using tide-gauge data

*All computations results presented in this and the following sub-sections are produced during the thesis research and can be considered as a part of the main contributions of this thesis work.*

The methodology for application of WMRA described in Chapter III and in Al-Kherayef et al. (2019a) has been applied to detect remove jumps/steps in TG records (see Figure 5.5) and to remove non-tidal effects in TG time series (see Figure 5.6. ). As a result of WMRA decomposition, de-noising and reconstruction, *a linear trend* in TG records time seris can be constructed and estimated according to methodology Chapter III. Also, The derived trend usually contains *local* tectonic, technogenic and TG sensor vertical movements, which need to be investigated and canceled out. The investigation of those effects is done by utilizing the TG deformation network described in Chapter IV. This trend will allow the estimation of precise *local* MSL (with regard to TGBMs) at every TG station at the initial epoch 2014.75 of KSA-VRF14. The graphs shown there represent the application of WMRA for jimp/step detection and removal together with filtering out the non-tidal effects in TG records.

### 3. Comparison of SST from SA&TG and SST from GNSS&TG

#### 3.1. Comparison per TG stations

The combination of ellipsoidal heights from SA and GNSS together with TG estimated local MSL allows the computation of SST/DOT in two independent ways, given the geoid height either from a local geoid model or from an EGM. The following comparison (Table 5.6) shows good agreement between SST/DOT from both approaches (SA&TG and GNSS&TG) except the TG station YANB where the shallow waters affect the data from SA. The SST/DOT estimations will be applied for the oceanic approach for geopotential shift determination and the improvement of vertical datum per every station.

### 4. Conclusions & recommendations

- The conclusions below confirm that the accuracy of both approaches corresponds to the accuracy of SA, GNSS and TG data assuming that the effect of EGM/Geoid is a constant per TG station. In such way the conclusions regarding accuracy of SST/DOT can be directly related to the accuracy of MSL ellipsoidal heighs determined by SA and GNSS ellipsoidal heights per TGBMs.
- The SST differences between SA&TG and GNSS have a mean value of 1.6 cm with an accuracy of 4.5 cm, which corresponds to expected accuracy of satellite altimetry data;
- The accuracy of Gulf area (4-5 cm) is better than Red Sea (7-8 cm) explained by much smaller area coverage;

### 3.2. Comparison per coastal regions

Tables 5.6 and 5.7. show estimates and statistics for both approaches and their differences per coastal regions:

- Good agreement between mean values for SSTs from both approaches: Gulf 0.1 cm, Read Sea  2.8 cm and 1.6 cm for entire KSA coast;
- The average difference between Gulf and Read sea MSL is ~7 cm;
- Satellite Altimetry together with TG records can provide a reliable estimate of MSL ellipsoidal heights assuming the effect of coastal conditions and the EGM utilized;
- The final estimate of SST/DOT or corresponding ellipsoidal heights can be determined as weigthed mean of the two approaches, combining their advantages – *no dependence* of SA data from local movements and better pointwise accuracy of TG data in shalow coastal areas.

- ***Final conclusions***

  The following conclusions can be drawn about SST/DOT determination by two independent approaches:

  - The derived SST/DOT can be seccefully applied in the oceanic approach for geopotential shift and corresponding geopotential numbers determination per every TG station.
  - *The utilization of those geopotential numbers can lead to a significant improvement of VD determination per every TG station.*
  - The estimated geopotential numbers (geopotenial values) based on SST/DOT computed above can be used for final solution of multi VD problem described in the next Chapter section.

**C. Solutions for new regional multi-Vertical Datum Problem**

As per Chapter IIIF, the following details regarding the main research task of this thesis are presented for completeness of the Chapter according to Al-Kherayef et al. (2019b):

THE MAIN RESEARCH QUESTION IS: How to combine different geodetic earth observations for improvement and definition of a Vertical Reference Frame based on the approach for unification of Local Vertical Datums (LVDs)

The main characteristics of the newly proposed problem are

- Unification of several LVDs will be applied for one large regional VRF definition based TG Network spanned on two water basins – Red Sea and Arabian Gulf;
- All three basic strategies usually applied separately will be implemented together, taking advantage of the fact that KSA has established in a relatively short time all necessary geodetic networks with high accuracy

requirements; Also, KSA has collected (or very soon will collect) all data and information necessary for simultaneous application of all strategies described above;

- Usually, unification is applied on several LVDs and each one is fixed to 1D Vertical datum at specific TG station;
- BUT in the new problem the question is how to produce ONE Multiple LVDs' Regional Vertical Datum based on several TG stations linked through many different types of data and information.

DEFINITION*: Global Vertical Datum Problem*: A 3D vertical datum is an equipotential gravity field surface with assigned a constant geopotential value $W_0$, which serves as the reference surface for physical heights.

Traditionally, a local vertical datum is based on the equipotential surface of the gravity field of the Earth that coincides with an idealized mean no-motion surface of the oceans. In such case, ${W_0}^J$ is the geopotential of the mean sea level at the origin of a single levelling network noted as $J$, or many different local, national and regional LVDs exist. However, a single unified and global vertical datum is required to provide a reference surface for monitoring the solid and fluid Earth and enabling the connection between the geometric and gravity reference frames

Table 5.6 Comparison between SST/DOT from both approaches (SA&TG and GNSS&TG)

| TGID | SST (SA&TG) [m] | SST (GNSS&TG) [m] | Difference [cm] |
|---|---|---|---|
| **ABUQ** | 0.458 | 0.419 | -3.9 |
| **OQAR** | 0.556 | 0.513 | -4.2 |
| **TANO** | 0.486 | 0.494 | 0.9 |
| **MARD** | 0.413 | 0.482 | 6.9 |
| **KHAF** | 0.531 | 0.537 | 0.6 |
| **JIZA** | 0.621 | 0.554 | -6.7 |
| **QUNF** | 0.607 | 0.625 | 1.8 |
| **JEDD** | 0.654 | 0.634 | -1.9 |
| **YANB** | 0.522 | 0.440 | -8.3 |
| **WAJH** | 0.604 | 0.541 | -6.3 |
| **DUBA** | 0.432 | 0.419 | -1.3 |
| **MAGN** | 0.547 | 0.580 | 3.4 |

Regionally, unified datums are required for international engineering projects, flooding control, coastal hazard studies, unification of national gravity anomaly databases, and improvement of continental geoid models. The problem of the unification of LVDs is referred as a Vertical (or Height) Datum (VD/HD) problem.

DEFINITION OF A NEW REGIONAL MULTI-VERTICAL DATUM PROBLEM (m-VDP): To define a m-VDP based on the geopotential offsets $\delta W_0{}^N$ at $N$ tide-gauge stations with regard to a global geopotential value $W_0$, determined by all types of available geodetic observations and computation strategies.

Table 5.7 Comparison per coastal regions from both approaches (SA&TG and GNSS&TG)

| REGION | SST (SA&TG) [m] | SST (GNSS&TG) [m] | Difference [cm] |
|---|---|---|---|
| **ARAB GULF** | | | |
| **mean** | **0.489** | **0.489** | **0.1** |
| obs. std | 0.057 | 0.044 | 4.5 |
| mean.std | 0.026 | 0.020 | 2.0 |
| **RED SEA** | | | |
| **mean** | **0.570** | **0.542** | |
| obs.std | 0.075 | 0.084 | 4.5 |
| mean.std | 0.028 | 0.032 | 1.7 |
| **Difference between Red Sea and Gulf** | **0.081** | **0.053** | |
| **Red Sea and Gulf** | | | |
| **mean** | **0.536** | **0.520** | **-1.6** |
| obs.std | 0.078 | 0.073 | 4.5 |

## 1. Strategies for m-VD problem solutions

### 1.1 Strategy 1: A direct connection by levelling, gravimetry and GNSS/Leveling data - determination of $W_0$ shift of GNSS /Leveling or local gravimetric geoid from a global geopotential model

#### 1.1.1. Mean Datum Offsets of Local Vertical Datum with regard to Global Vertical Datum

The mean datum offset $\delta Np$ of GNSS/Levelling derived geoid with regard to GGM. The total number of BMs with GPS/GNSS/Levelling data is 3477 utilized to produce an improved KSA-GEOID according to Grebenitcharsky et al. (2005) and Rangelova & Sideris (2012). After testing different GGMs, two GGMs have been utilized - XGM2016 as a most recent GOCE based combined model and EIGEN5C1 as an old intermediate type of GGM to investigate possible differences coming from imperfection of GGMs applied.

*No significant differences exist between one of the most recent GGM and an old version of EIGEN EGMs. The estimations for XGM2016 can be accepted as offsets between the GPS/GNSS/Levelling Local Geoid and the Global Vertical Datum and are equal to:*

- $\delta N = 0.221\ [m]$ — the offset in Local Geoid from the global EGM;
- $\delta W = -2.21\ [m^2/s^2]$ — the offset in geopotential values of Local Geoid with regard to the global EGM;
- $W_{LVD} = \mathbf{62636853.79}\ [m^2/s^2]$ — **the geopotential value for the Local Geoid**

Table 5.8 Estimations of the offsets in geoid height/geopotential values with regard to GGMs.

| | **XGM2016** | **EIGEN5C1** | Difference |
|---|---|---|---|
| $\delta N$ [m] | 0.2211 | 0.2149 | 0.0062 |
| $\delta W\ [m^2/s^2]$ | -2.2114 | -2.1600 | 0.0514 |

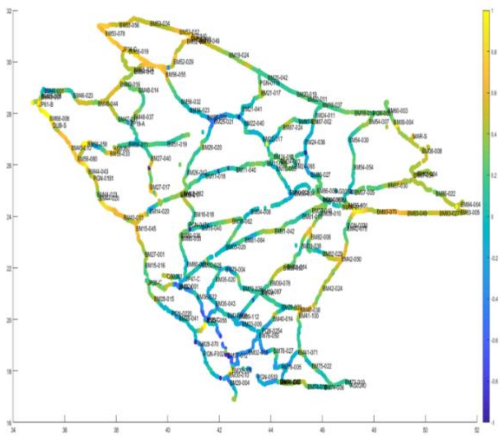


Figure 5.7. GPS/GNSS/Levelling Geoid heights per ~3500 BMs used for computation of $\delta N\ [m]$ and $\delta W\ [m^2/s^2]$: Colure range: -1:1 meters

## 1.2 Strategy 2: Tide Gage Record, Satellite Altimetry & GNSS data Analysis for Mean Sea Level and its trend determination

This subsection presents the results of numerical test following the Strategy 2, described in Chapter III.

### 1.2.1. Datum Offsets of Local Vertical Datum with regard to Global Vertical Datum per TG station

Table 5.9 The main sources of data for computing the datum offsets per TGs. Units: [m]:

| TGID | ELL.H | WL TG R | $R_{corr}$ | SST (MT) | SST (FT) | $H_{orth}$ (MT) | $H_{orth}$ (FT) | KSA-GEOID |
|---|---|---|---|---|---|---|---|---|
| **ABUQ** | -27.99 | 2.05 | 2.03 | 0.61 | 0.58 | 2.12 | 2.01 | -30.11 |
| **OQAR** | -25.13 | 1.51 | 1.44 | 0.62 | 0.64 | 1.53 | 1.48 | -26.72 |
| **TANO** | -23.10 | 2.18 | 2.19 | 0.61 | 0.55 | 2.27 | 2.13 | -25.35 |
| **MARD** | -19.69 | 2.50 | 2.43 | 0.62 | 0.63 | 2.52 | 2.46 | -22.28 |
| **KHAF** | -14.47 | 2.32 | 2.38 | 0.57 | 0.50 | 2.43 | 2.29 | -16.82 |
| **JIZA** | -2.50 | 1.62 | 1.53 | 0.55 | 0.61 | 1.55 | 1.54 | -4.12 |
| **QUNF** | 2.66 | 2.08 | 2.17 | 0.50 | 0.46 | 2.14 | 2.03 | 0.63 |
| **JEDD** | 6.69 | 1.82 | 1.82 | 0.53 | 0.60 | 1.82 | 1.82 | 4.89 |
| **YANB** | 10.81 | 2.20 | 2.05 | 0.69 | 0.72 | 2.21 | 2.17 | 8.47 |
| **WAJH** | 13.45 | 1.75 | 1.81 | 0.59 | 0.52 | 1.87 | 1.73 | 11.65 |
| **DUBA** | 15.96 | 1.94 | 1.76 | 0.70 | 0.75 | 1.94 | 1.91 | 13.86 |
| **MAGN** | 16.54 | 1.89 | 1.97 | 0.55 | 0.49 | 2.00 | 1.87 | 14.64 |

**Table 5.9 abbreviations:**

ELL.H    Ellipsoidal height in KSA ITRF2000, epoch 2003.1998
WL TG R    TG reading from WMRA referenced to TGBM at 2014.75
$R_{corr}$    Corrected for vertical movements TG records R from TG monitoring project
SST(MT)    Sea surface topography in mean tide system
SST(FT)    SST in Free Tide System according to Ekman (1989)
$H_{orth}$ (MT)    Orthometric Height in Mean Tide System
$H_{orth}$ (FT)    Orthometric Height in Free Tide System
KSA-GEOID    Gravimetric KSA Geoid 2017

For validation of orthometric heights $H_{orth}$ of TGBMs derived from SA & TG and GNSS & TG data a least-squares adjustment has been realized on KSA-NVN by fixing all TGBM orthometric heights. The results and their analysis are presented below:

*Validation scenario 1 – differences in orthometric heights:*

Analyzing the differences between Jeddah fixed solution and the LSA solution with all TG station heights fixed the following conclusions are available:

- General slope is visible from NW to SE direction from -0.025 m to -0.080 m
- Expected small offsets around Jeddah TG up to -1 cm
- Not expected offsets up to -8 cm in SW
- All offsets have systematic behavior and show that possible improvement is expected. The difference between Jeddah and Ras Tanura TGBM heights is 4-5 cm for both solutions but it has a systematic effect and is close to SST differences between both TG *Validation scenario 2 – differences.*

- *The formal accuracy estimation for KSA-VRF 14 is purer by 4 cm w.r.t to new combined solution especially in TGs remotely located from Jeddah TG;*
- *Expected small formal errors between TG stations up to 1cm, located in between the Red Sea and Arabian Gulf and around;*
- *The final formal accuracy is increasing directly in the coastal area in vicinity of every TG station, while in the internal territory of KSA its magnitude is no greater than 1 cm.in standard deviations.*

- The last finding supports the idea that there is no significant difference between MSLs in both TG stations which significantly contradicts with the common opinion in the past that MSLs between Ras Tanura (Arabian Gulf) and Jeddah (Red Sea) differ about **79** cm. The last conclusion can be considered as *a significant contribution* in clarifying the existing significant offset in the past between Red Sea and Arabian Gulf MSLs.

- It can be considered as result of deficiency in precise leveling procedure applied at that time which did not take into account any refraction and orthometric corrections (based on real gravity values) per KSA-NVN BMs stations.

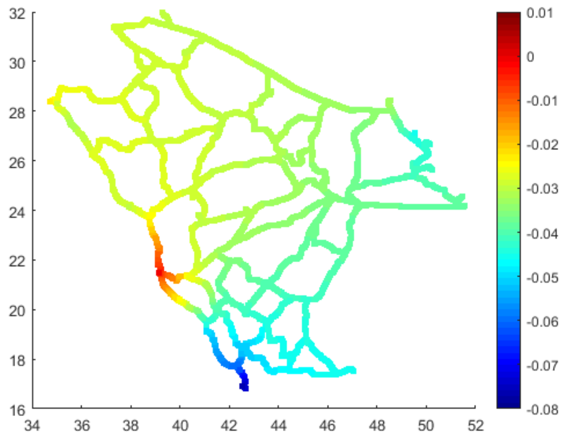



Figure 5.8. Offsets of KSA-VRF14 (fixed Jeddah TG) heights w.r.t. to combined solution (all TG fixed) heights from TG&SA analysis:. Units:[m]

*Validation scenario 2 – differences in standard deviations*

This scenario is presented in details in the thesis and confirms the results from scenario 3 in terms of formal accuracy estimation. It is also part of sub-section 1.3. representing a combined solution.

*Validation scenario 3 – ratio between the formal errors*

- The formal error estimate for KSA-VRF 14 is 5 times greater (less accurate) than the new combined solution, especially in areas in vicinity of TGs.
- The rest of the area's formal errors for KSA VRF14 solution are 1.5 – 2 times higher (less accurate) than the new solution.
- The results of new LSA confirm the expected improvement of the BM orthometric heights accuracy after fixing the heights of TGBMs - more than 2 times in coastal areas and 1.5 – 2 times inside the KSA.
- Final conclusions for validation scenario 3

The following important final conclusions are possible:

- *Those results are an indirect proof that the estimated orthometric heights per TGBM by combination of different types of data are very close to the expected real values,* otherwise any significant deviations from their real values would cause the standard deviations to increase (according to the law of random errors distribution) in the middle of KSA, which itself means deterioration of accuracy.
- *Now, the final steps are:*
  - to combine all 3 Scenarios for geopotential offset determination per every TG station in order to estimate their geopotential values $\boldsymbol{W_j}$ per $J^{th}$ – TG Station;
  - to perform an *LSA* in terms of geopotential height differences of entire KSA-NVN by fixing $\boldsymbol{W_j}$ per every TGBM.

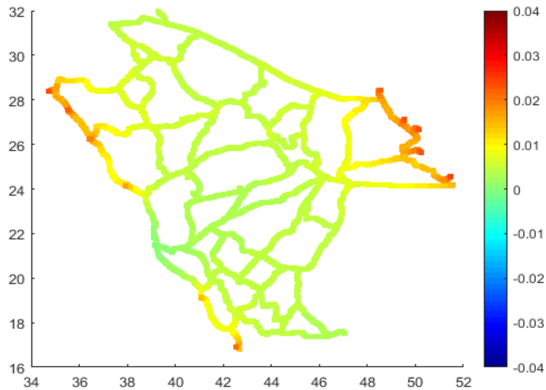



Figure 5.9. Differences of KSA-VRF14 (fixed Jeddah TG) standard deviations (formal errors) with regard to standard deviations of combined solution(all TG fixed) heights from TG&SA analysis. Units: [m]

### 1.3 Combination of Strategies 1-3: Combination of all available strategies and data for geopotential offsets and values determination and their utilization for solution of mVDP

This sub-section represents the results of combining strategies from 1 to 3 and data, which could be considered as ***a combined solution of the mVDP***.

#### 1.3.1 Utilization of a modified Scenario 3 for solution of mVDP solution

The Scenario 3 - the geodetic boundary value problem (GBVP) approach – is connecting the datums by solving the GBVP and computing LVD offsets with respect to a global equipotential surface (GGM). A pre-requisite condition to apply this scenario over KSA is to have homogeneous gravity data around every TG station which can be realized after finalization of KSA-Airborne gravity observation project. Since the main goal of this scenario is to determine the local geoid height per every TGBM using dense gravity data in the vicinity of every TG station (in radius of 50-100 km around, depending on the reference GGM used). In order to demonstrate the combination of Scenarios 1-3, Scenario 3 needs to be modified in the following way:

- Assuming TGBMs geoid heights to be interpolated from the most recent *gravimetric geoid* GRAV-GEOID17 over KSA (local VRF related to the local geoid) or

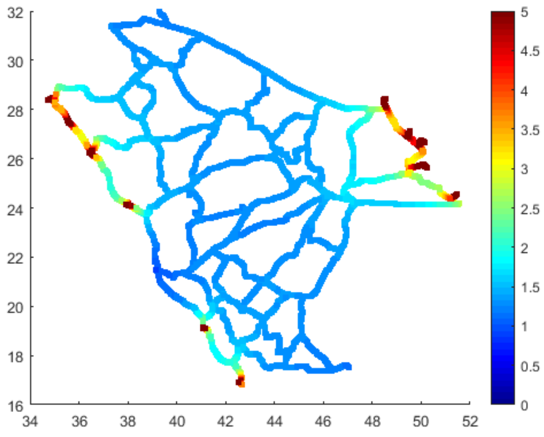



Figure 5.10. Ratio of formal errors from KSA-VRF14 (fixed Jeddah TG) heights over the formal errors of combined solution (all TG fixed) from TG&SA analysis. Units: [no units, just ratio]

- Assuming TGBMs geoid heights to be interpolated from the most recent *hybrid geoid* KSA-GEOID17 or updated by new GNSS determined ellipsoidal heights per TGBM (local VRF related to MSL at TGBM B in Jeddah).

Assuming the geoid height per TGBM is result of a regional solution of GBVP using Fast Fourier Transforms (FFT) in planar domain the modified Scenario 3 can be applied together with Scenario 1 & 2 to determine the geopotential offset and geopotential value per every TGBM and applying an LSA KSA-NVN with fixed geopotential numbers or Geo-potential values per TGBM.

### 1.3.2 Combination of all available data & information together with all three Scenarios 1-3 for geopotential number / geopotential value estimation per TGBM

The block diagram of combining data and scenarios for computing geopotential numbers ($C_J$ ) and geopotential values ($W_J$) per TGBM *J*. is presented below for a brief explanation of the procedure for combination of data and scenarios.

## 2. Analysis of results of numeric experiment for combining all data and scenarios

### 2.1. Analysis of computed geo-potential values per TG station

The following results show the final *geopotential numbers of MSLs* per all TGBMs, both from SA and from TG analysis, combining all available data types and the three Scenarios assuming geoid heights in Scenario 3 to be equal to Local Hybrid Geoid (as regional solution of Stokes GBVP) over entire KSA.

Such experiment can be considered as an additional check and should lead to Helmert orhometric heights similar to the ones determine by LSA. This result has been discussed in previous subsections. Also, the following important conclusions are available:

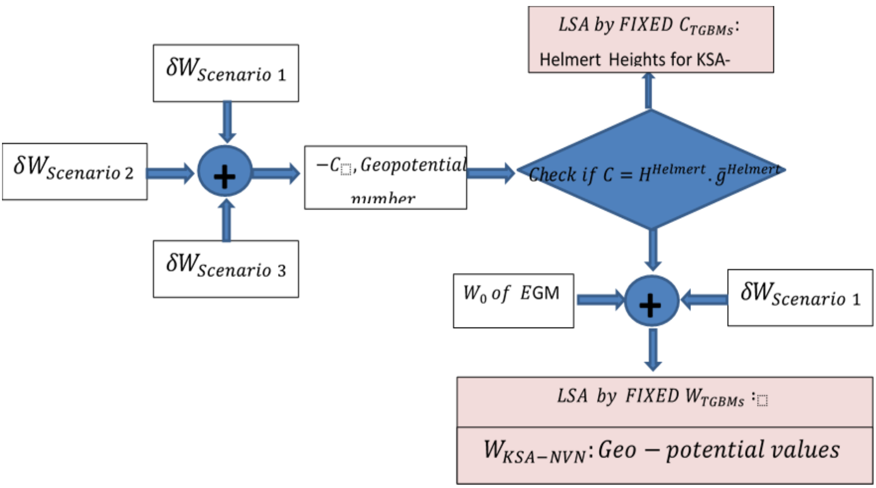



Figure 5.11. Block diagram for combining scenarios 1-3 for computing geopotential values and orthometric heights of KSA-NVN BMs

- The geopotential offset *ΔW* from Tide Gauge (TG) and Satellite Altimetry (SA) analysis has been computed with regard to the local KSA-GEOID17 (which itself is result of regional solution of Stokes's GBVP in terms of Scenario 3);
- Utilizing $\delta W$ = *2.21* $m^2/s^2$ from Strategy 1 for the offset of KSA-GEOID2017 with regard to GGM XGM2016 ($W_0$ = *62636856.0*), the *Wj* per every TG station has been computed both for TG and SA;
- The weighted mean is result of assuming accuracy of SA to be twice less than TG accuracy;
- The differences between combined geopotential solution and the one from Scenario 2 except for ABUQ TG are within the observation accuracy of both – SA and TG approaches;
- The mean values *ΔW* from *combined solution* per KSA, Red Sea and Arab Gulf show differences between SA & TG:

| | | |
|---|---|---|
| KSA: 0.03 g.p.u.<br>KSA: 3.6 cm | Red Sea: 0.04 g.p.u<br>Red Sea: 4.6 cm | Arab Gulf: -0.02 g.p.u.<br>Arab Gulf: -2.4 cm |

Table 5.10 Geopotential numbers of MSLs per all TGBMs. Units [$m^2/s^2$]

| UNITS: m^2/s^2 | cos(lat) weighted | $W_{msl}$ - $W_{local/SA}$ | cos(lat) weighted | $W_{msl}$ - $W_{local/TG}$ |
|---|---|---|---|---|
| ***W0 = 62636853.8*** | **-1.3** | **-1.5** | **-0.3** | **-0.4** |
| **GULF** | **-0.9** | **-1.0** | **-0.3** | **-0.3** |
| Ras Tanura TG | **-0.3** | **-0.3** | **-0.3** | **-0.3** |
| | **0.3** | **0.4** | **-0.2** | **-0.2** |
| | **-0.7** | **-0.8** | **-0.7** | **-0.8** |
| **READ SEA** | **-1.6** | **-1.6** | **-0.6** | **-0.7** |
| | **-1.4** | **-1.5** | **-1.3** | **-1.4** |
| Jeddah TG | **-1.8** | **-2.0** | **-1.5** | **-1.6** |
| | **-0.6** | **-0.7** | **0.2** | **0.3** |
| | **-1.3** | **-1.5** | **-0.7** | **-0.8** |
| | **0.2** | **0.2** | **0.4** | **0.4** |
| | **-0.8** | **-0.9** | **-1.0** | **-1.1** |

- The mean values *ΔW* from *Scenario 2* per KSA, Red Sea and Arab Gulf show *very similar differences* to the combined solution between SA & TG:

| | | |
|---|---|---|
| KSA: 0.02 g.p.u.<br>KSA: 2.3 cm | Red Sea: 0.04 g.p.u<br>Red Sea: 3.5 cm | Arab Gulf: -0.03 g.p.u.<br>Arab Gulf: -2.8 cm |

- The difference between weighted geopotential values between Ras Tanura and Jeddah (from JEDD-S TG to TANO-S TG) MSL is estimated to be ***+0.14 g.p.u.***, which corresponds to an offset of ***-14.3 cm***.
- The last estimated offset is result of combination of many different observations and it significantly differs from the OLD offset from the old Jeddah 69 vertical datum to the SVD78 datum assumed to be ***+79.0 cm*** in the pass. Such significant difference can be explained to be due to the fact that in the old precise leveling procedure orthometric height corrections have not been applied or do not use real gravity values and NO refraction corrections have been applied.

Table 5.11 Geopotential values of MSLs per all TGBMs. Units $[m^2/s^2]$:

| UNITS: m^2/s^2 | TG ID | Wj SATELLITE ALTIMETRY | Wj TIDE GAUGE | $W_{TG}$-$W_{SA}$ m^2/s^2 | $W_{TG}$-$W_{SA}$ [cm] | $SST_{SA}$-$SST_{TG}$ [cm] | δW-δSST [cm] | WEIGTHED MEAN SA&TG [m^2/s^2] |
|---|---|---|---|---|---|---|---|---|
| ***W0 = 62636858.2*** | **ABUQ** | 62636852.3 | **62636853.4** | 1.1 | 11.2 | 3.7 | 7.5 | 62636853.0 |
| **GULF** | **OQAR** | 62636852.8 | **62636853.5** | 0.7 | 7.0 | 4.0 | 3.0 | 62636853.2 |
| Ras Tanura TG | **TANO** | 62636853.5 | **62636853.4** | 0.0 | -0.2 | -1.1 | 0.9 | 62636853.4 |
| | **MARD** | 62636854.2 | **62636853.5** | -0.6 | -6.4 | -7.1 | 0.7 | 62636853.8 |
| | **KHAF** | 62636853.0 | **62636852.9** | -0.1 | -0.8 | -0.8 | 0.0 | 62636853.0 |
| **READ SEA** | **JIZA** | 62636852.1 | **62636853.1** | 1.0 | 10.0 | 6.5 | 3.5 | 62636852.8 |
| | **QUNF** | 62636852.3 | **62636852.4** | 0.1 | 0.8 | -2.0 | 2.8 | 62636852.3 |
| Jeddah TG | **JEDD** | 62636851.8 | **62636852.2** | 0.4 | 3.8 | 1.7 | 2.1 | 62636852.1 |
| | **YANB** | 62636853.1 | **62636854.0** | 0.9 | 9.6 | 8.1 | 1.5 | 62636853.7 |
| | **WAJH** | 62636852.3 | **62636853.0** | 0.7 | 7.4 | 6.1 | 1.3 | 62636852.8 |
| | **DUBA** | 62636854.0 | **62636854.2** | 0.2 | 2.4 | 1.1 | 1.3 | 62636854.1 |
| | **MAGN** | 62636852.9 | **62636852.7** | -0.2 | -2.0 | -3.6 | 1.6 | 62636852.7 |

Table 5.12 Geopotential values & differences of MSLs per regions. Units $[m^2/s^2]$:

| REGION/TGs | Statistics | $W_{TG}$-$W_{SA}$ m^2/s^2 | $W_{TG}$-$W_{SA}$ [cm] | $SST_{SA}$-$SST_{TG}$ [cm] | δW-δSST [cm] | WEIGTHED MEAN SA&TG [m^2/s^2] |
|---|---|---|---|---|---|---|
| **RED SEA & GULF** | **mean** | 0.3 | 3.6 | 2.3 | 1.3 | 62636853.1 |
| | **std** | 0.5 | 5.5 | 4.5 | 2.0 | 0.6 |
| | | | | | | |
| **GULF** | **mean** | 0.2 | 2.2 | 0.6 | 1.5 | 62636853.3 |
| | **std** | 0.7 | 6.9 | 4.5 | 3.1 | 0.3 |
| | | | | | | |
| **RED SEA** | **mean** | 0.4 | 4.6 | 3.5 | 1.1 | 62636852.9 |
| | **std** | 0.4 | 4.6 | 4.5 | 0.9 | 0.7 |
| | | | | | | |
| **GULF-REDSEA** | **mean** | -0.2 | -2.4 | -2.8 | 0.4 | 0.3 |
| | **std** | 0.8 | 8.3 | 6.4 | 3.2 | 0.8 |
| | | | | | | |
| **TANO-JEDD** | | -0.4 | -4.1 | -2.8 | -1.3 | 1.4 |

### 2.2. Validation of computed geopotential values, numbers and orthometric heights trough LSA of observed geopotential height differences

Conducting LSA over entire geopotential height differences KSA-NVN by fixing geopotential numbers w.r.t. to MSL in Jeddah TG station (TGBM-B) at epoch 2014.75 can provide *an improved version of KSA-VRF14* which takes into account all available data, information and scenarios. It will allow a validation of LSA results with regard to existing KSA-VRF14 geopotential numbers and Helmert orthometric heights. *As a result, an improved version of KSA-VRF14 can be realized as solution of mVDP for the entire Kingdom of Saudi Arabia Territory.*

#### 2.2.1. General statistics of the LSA Network Adjustment

The following graph and general statistics show the LSA performance over KSA-NVN:

- In both cases – 1) Jeddah TGBM g.p.u. fixed and 2) All TGBMs g.p.u. fixed - the a priori standard deviation per $\sqrt{S[km]}$ for single run precise leveling section drops from 1.541 mm to 0.929 mm (case 1) and 0.934 mm (case 2)

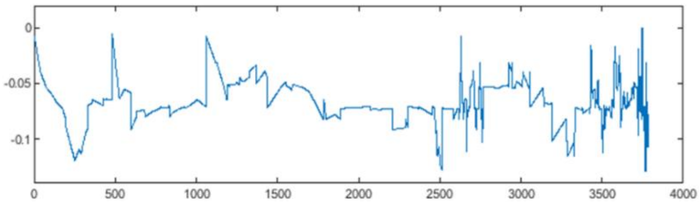



Figure 5.12. Differences between two cases - graphical distribution over the KSA Units:[m]

Table 5.13 LSA results over KSA-NVN. Units [m]:

```
LEVELING NETWORK ADJUSTMENT PROGRAM
ALL TGBM GEO-POTENTIAL NUMBERS FIXED
 APRIORI VARIANCE FACTOR :  1.000
 APOSTERIORI VAR. FACTOR :  0.873
 APRIORI      STD PER P=1 :  1.541
 APOSTERIORI STD PER P=1 :  0.934
```

```
LEVELING NETWORK ADJUSTMENT PROGRAM
JEDD TGBM-B GEO-POTENTIAL NUMBER FIXED
 APRIORI VARIANCE FACTOR :  1.000
 APOSTERIORI VAR. FACTOR :  0.863
 APRIORI      STD PER P=1 :  1.541
 APOSTERIORI STD PER P=1 :  0.929
```

- The a posteriori standard deviations for both cases are very similar which means fixing all TGBM heights does not introduce major distortions in the KSA-VRF14 (0.005 mm $\sqrt{S[km]}$) or 0.25 mm per 2500 km (1:$10^{-10}$ relative error);The following table and graphs represent the differences between two cases, their statistics and their graphical distribution over the KSA:
- There is a systematic shift of ~ 7 cm with regard to KSA-VRF14 vertical datum. The standard deviation per difference is ~2 cm which is within the tolerances of the provisional KSA-NVN accuracy at 700 km from the initial Jeddah TGBM-B. This shift needs a careful study to be conducted in future;

Table 5.14 Statistics of differences between two cases. Units [m]

| Number of BMs | MIN [m] | MAX [m] | MEAN [m] | STD [m] | RMS [m] |
|---|---|---|---|---|---|
| 3790 | -0.129 | 0.000 | -0.067 | 0.019 | 0.070 |

- There is a clear, up to -12.9 cm geographical distribution of the differences from KSA-VRF14 mostly around Arab Gulf TG stations in the Red Sea coastal areas. This offset coincides with the one derived in 2.1.
- *Since it is based on LSA of entire KSA-NVN it can be assumed that the difference between MSLs at JEDD TGBM and TANO TGBM is equal to* ***-0.129 m*** *decreasing from Red Sea to Arabian Gulf.*
- After the LSA, the mean values of differences between case 1 and case 2 with regard to MSL at Jeddah TG are:

| | | |
|---|---|---|
| RED SEA:**-0.069 m** | GULF:**-0.109 m** | Difference:**-0.040 m** |

- FINAL CONSLUSION: Summarizing the above conclusions about the new solution of mVDP over KSA it has an average shift from KSA-VRF14 of **-0.067 m** (see statistics above) and an slope from Red Sea to Arabian Gulf of **-0.040 m**. Additional study with independent data source (coming up new geoid based on airborne gravity data) could confirm the reliability of an improved realization of KSA-VRF14 based on LSA of geopotential differences and fixed geopotential numbers of MSLs at all TG stations.
    - It is interesting to find out that the averaged shift of *-0.067 m* above coincides very well with the half of the difference between *MSLs at JEDD TGBM and TANO TGBM (– 0.0645 m) or it can be concluded that:*
    - *The multi-vertical datum of* the new ***mVDP*** solution is located in the middle of both MSLs at Jeddah and Ras Tanura;
    - ***The last finding justifies the application of averaged vertical datum in the past (from practical point of view);***
    - ***And again confirms the reliability of the definition of new mVDP and its solution with regard to theoretical and practical developments of Vertical Datum concepts in the past.***

# Chapter VI: Conclusions, Recommendations and Contributions

## *A. Conclusions, recommendations and contributions*

### 2.Important research contributions

The following section presents the major thesis research contributions accomplished during the entire research duration. They are grouped in three major categories.

## 2.1. Contributions related to the developed methodology for improved vertical reference frame and vertical datum: Case study for KSA-VRF & KSA-VD

The following contributions would have a significant impact on future definitions of local, national and regional vertical reference frames and datum (see chapter VI, section A.1.1.):

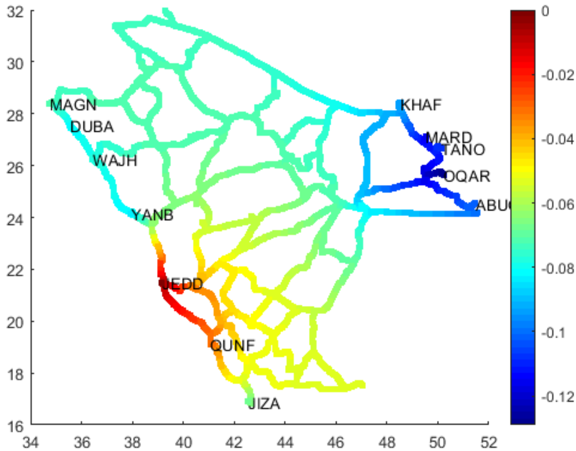



Figure 5.13. Geographical distribution of differences between two cases over the KSA Units:[m]

i. An adequate, reliable and achievable concept has been formulated and proved by a numerical experiment - *'Combination of different types of collocated geodetic observations at a TG station would allow the computation of its geopotential values/numbers and the orthometric heights of up to the highest possible accuracy requirements.'*

ii. An important contribution consists of detailed recommendations how the developed methodology can be efficiently utilized for geopotential numbers/orthometric heights and their accuracy estimations for improvement of the current KSA-VRF14 over the entire KSA-NVN;

iii. Achieving the following detailed research objectives is also considered as an important contribution which can improve any local/national/regional VRF and VD by:

   - An improvement of 70% of accumulated misclosures (between forward and backward measurements) and more than 50% improvement of KSA-NVN loop misclosures are considered as a significant contribution in precise leveling observations processing

- An important contribution is the DOT/SST improvement per TG station by utilizing TG record data, Satellite Altimetry data, GNSS ellipsoidal heights and the most recent Global Geopotential Models (GGMs). The specific contributions are identified as:
  - *the average shift from Gulf to Read sea MSL is determined to be ~7 cm;*
  - *the SST differences between SA&TG and GNSS have mean value of 1.6 cm with an accuracy of 4.5 cm, which corresponds to expected accuracy of Satellite Altimetry;*
  - *the improved SST/DOT, applied later in the oceanic approach, is leading to improved geo-potential shift and corresponding geo-potential values/numbers per every TG station.*

## 2.2. CONTRIBUTIONS RELATED TO THE DEFINITION OF NEW m-VDP AND FINDING ITS NUMERICAL SOLUTION

The following are considered as scientific and research contributions of the definition of new m-VDP and finding its numerical solution (see CHAPTER VI, section A.1.2.):

*iv.* The definition of new multi-Vertical Datum Problem*: to determine geo-potential offsets $\delta W_0^N$ at N number of TG stations with regard to a Global Geopotential value $W_0$, determined by all types of available geodetic observations and computation strategies;*

v. The proof of m-VDP definition reliability by results of numerical experiment;

vi. As specific contributions are considered the estimations of offsets with regard to XGM2016:
- $\delta N$ = 0.226 [m] *the offset of Local Geoid in meters;*
- $\delta W$ = -2.210 [$m^2/s^2$] *the offset of Local Geoid in geopotential values;*
- $W_{LVD}$= 62636853.79 [$m^2/s^2$] *the geopotential value for the Local Geoid.*

**vii. THE MAIN RESEARCH CONTRIBUTION - a final numerical solution of m-VDP;**

- Based on *combination of all 3 Scenarios for geopotential offset determination per every TG station;*
- Presented by:
  - *The geopotential values per every TG Mean Sea Level (Table 5.1)*
  - *Differences between KSA-VRF14 and m-VDP solution together with their statistics (Fig. 5.8 and Fig. 5.9 a,b)*
  - *Differences (Fig. 5.11 & Fig. 5.12) with their statistics (Table 14) between two cases - the effect of constrained geopotential values at every TGBM;*

- Main contribution performance:
  - *An average shift of new solution from KSA-VRF14:* ***-0.067 m*** *with a slope from Red Sea to Arabian Gulf:* ***-0.040 m****;*
  - *Good reliability and performance of the solution of m-VDP: the averaged shift above coincides very well with the half of the difference between MSLs at JEDD TGBM and TANO TGBM or the new m-VDP solution is situated in the middle of both MSLs;*
  - *the new m-VDP solution justifies the application of averaged vertical datum in the past and confirms the reliability of the new m-VDP definition and its solution with regard to theoretical and practical developments of Vertical Datum concepts in the past.*
  - *Additional study with independent data source (coming up new geoid based on airborne gravity data) could confirm the reliability of an improved realization of KSA-VRF14 - recommendation.*

viii. FINAL TASK CONTRIBUTION: proposal for possible improvement of KSA-VRF14 - an improved version of KSA-VRF14 can be realized based on disseminating the solution of m-VDP over the entire Kingdom of Saudi Arabia Territory by constrained Least-Squires Adjustments investigating two options:

- constraining computed geopotential numbers of 11 TGBMs with regard to MSL in Jeddah TG station (TGBM-B) at epoch 2014.75 (keeping fixed MSL at Jeddah TG). ***It provides an improved replica of KSA-VRF14i based on all available data, information and scenarios;***

- LSA over all geopotential height differences of KSA-NVN by constraining computed geopotential numbers (g.p.n.) of all 12 TGBMs. ***It provides a NEW version of KSA-VRF based on all available data, information and scenarios;***

### 2.3. CONTRIBUTIONS RELATED TO IMPORTANT FINDINGS RELATED TO THE RESULTS OF NUMERICAL SOLUTION OF m-VDP

Some very important findings as secondary outputs will be presented below regarding the link between the old KSA-VRFs: Jeddah1969, SVD78 and the current KSA-VRF14 together with its improved version KSA-VRF14**i** based on the solution of m-VDP.

ix. CONTRIBUTION OF STRATEGY 2 (SATELLITE ALTIMETRY, TIDE GAUGE AND GNSS): The MSL geo-potential difference from JEDD-S TG to TANO-S is **+0.14 g.p.u**, which corresponds to an offset of **-14.3 cm**;

x. CONTRIBUTIONS OF STRATEGIES 1-3 (ALL DATA AND SCENARIOS):

- There is a clear, up to -12.9 cm, geographical distribution of the differences w.r.t. KSA-VRF14 with standard deviation per difference is **~2 cm** which is in the limit of the provisional accuracy of KSA-NVN up to

700 km from the initial Jeddah TGBM-B. This shift needs a careful study to be conducted in future;

- The difference between MSLs from Jeddah to Ras Tanura is **-12.9 cm** decreasing from Red Sea to Arabian Gulf. It is located mostly around Arab Gulf TG stations in the coastal area and based on LSA of entire KSA-NVN;
- This MSL difference estimation coincides up to 1.4 cm with the one provided in STRATEGY 2.

xi. **THE MOST IMPORTANT SECONDARY CONTRIBUTION – NO SIGNIFICANT SHIFT IN MSLS OF RED SEA AND ARAB GULF:** The detected past significant offset (79 cm) between the MSL at Jeddah (Jeddah 69/SVD78) and the one at Ras Tanura TGs **ACTUALLY DOES NOT EXIST!** Such contribution is a significant breakthrough for KSA geodetic applications.

xii. **The final averaged Red Sea – Arab Gulf MSL offset assumed to be: -13.6 cm**

- It is a result of combination of many different observations and it significantly differs from the OLD offset from the old Jeddah 69 vertical datum to the SVD78 datum assumed to be **-79.0 cm** (introduced +79 cm offset between both height systems).
- Such significant difference can be explained to be due to the fact that in the old precise leveling procedure orthometric height corrections have NOT been applied (or not using real gravity values) and NO refraction corrections have been utilized.

The conclusions and contributions discussed in this thesis research could be considered in line with the entire global geodetic community efforts for transition:

- ❑ from static to dynamic/changing in time Vertical Reference Frames & Vertical Datums;
- ❑ from precise to ultra-high accurate Vertical Reference Frames & Vertical Datums;
- ❑ from local/national to regional/global coverage of Vertical Reference Frames & Vertical Datums – the improved KSA-VRF14 could become the core of ARABREF spatial reference frame planned to be released under UN-GGIM Arab states study group;
- ❑ from nation/regional Vertical Reference Frames & Vertical Datums to a unified International Reference Height System and an improved KSA-VRF based on new KSA-GEOID20 could be considered as an important realization of IRHS.

## 3.Presentations of important conclusions and research contributions

The conclusions and contributions described above are presented in the following publications:

**Al-Kherayef, O., V.Valchinov, R. Grebenitcharsky, S. Valcheva, B. Al-Muslmani, U. Al-Rubaia (2018),** *Refraction coefficient determination and modelling for the territory of the Kingdom of Saudi Arabia*, FIG CONGRESS Proceedings, May 6 – May11, 2018, Istanbul, Turkey

**Al-Kherayef, O., Vergos G, R. Grebenitcharsky (2019),** *Improvement of Vertical Datum by Utilizing Tide Gauge Records Analysis, Satellite Altimetry DATA and GPS/GNSS observed ellipsoidal heights : Case study for Kingdom of Saudi Arabia*, G02 - STATIC GRAVITY FIELD AND HEIGHT SYSTEMS, 27th General Assembly of IUGG, Montreal, Canada, July 08 – July18, 2019

**Al-Kherayef, O., R. Grebenitcharsky, M. Minchev (2019),** *Improvement of Kingdom of Saudi Arabia National Vertical Reference Frame definition by all available classical and new geodetic Earth observation data: A case study*, G02 - STATIC GRAVITY FIELD AND HEIGHT SYSTEMS, 27th General Assembly of IUGG, Montreal, Canada, July 08 – July18, 2019

**Al-Kherayef, O., R. Grebenitcharsky (2019),** *The Kingdom of Saudi Arabia Geodetic and Vertical Reference Frames – the main Components of National Spatial Reference System*, EUREF 2019 Symposium, Tallinn, Estonia, 23-25 May, 2019

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## Abbreviation index

ARAMCO – Arabian-American oil Company
BLUE - best linear unbiased estimation
BM – bench mark
BVP - boundary value problem
CORS - Continuous Reference Station
DORIS - Détermination d'Orbite et Radiopositionnement Intégré par Satellite
DOT – dynamic ocean topography
DVD - dynamic vertical datum
EGM – Earth gravitational model
EVRF - European Vertical Reference Frame
GBVP - geodetic boundary value problem
GCS – KSA General Commission for Survey
GEO - geodetic Earth observation
GGM - global geopotential model
GIS – geographic information system
GLN - Geodetic Leveling Network
GNSS – global navigation satellite system
GOCE - Gravity field and steady-state Ocean Circulation Explorer
g.p.n., g.p.u, - geopotential number, geopotential unit
GPS – United States of America’s Global Positioning System
GRACE - Gravity Recovery and Climate Experiment
GRF – geodetic reference frame
GVD – global vertical datum
HSU - height system unification
IRHS - International Reference Height System
KSA – Kingdom of Saudi Arabia
LSA – least squares adjustment
LVD – local vertical datum
MSL – mean sea level
m-VDP – multi vertical datum problem
NAD – North American Datum
NAP - Normaal Amsterdams Peil
NGI – national geodetic infrastructure
NVN - national vertical network
NTGN – national tide-gauge network
RMS – root mean square (error)

SA - satellite altimetry
SANSRS - Saudi Arabia National Spatial Reference System
SLR - satellite laser ranging
SST – sea surface topography
STD – standard deviation
TG – tide-gauge
TGN – tide-gauge network
TGBM – tide-gauge bench mark
UELN – Unified European Leveling Network
VD – vertical datum
VDP - vertical datum problem
VLBI – very-long base interferometry
VRF – vertical reference frame
WMRA - Wavelet Multi-Resolution Analysis