COMPUTER SIMULATION OF SALT CAVERN LEACHING PROCESS

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ABSTRACT. Underground caverns in rock salt are being constructed by leaching of salt deposits through the tubing. This complex development is composed of different physical, chemical and thermodynamical processes and may be described by a system of non-stationary multidimensional partial differential equations which cannot be solved analytically (Каратыгин *et al* , 1994). That is why to solve such equations and thus to get technological characteristics of leaching it is necessary to use numerical methods. This paper presents the mathematical model of laminar isothermal flow of incompressible two-component liquid inside the cavern during leaching process. The mathematical model and related computational algorithm are supported by a computer program which is able to calculate main features of technological process and to demonstrate development of cavern in time.

КЪМПЮТЪРНО СИМУЛИРАНЕ НА ПРОЦЕСА ИЗЛУЖВАНЕ В СОЛНИ КАВЕРНИ

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РЕЗЮМЕ. Подземните каверни в сол-съдържащите скали са изградени за излужване на солните залежи чрез тръбопроводи. Тази метод включва комплекс от физични, химични и термодинамични процеси и може да бъде описан чрез система от променливи, многомерни диференциални уравнения, които не могат да бъдат решени аналитично. Това е така защото за да се решат подобни уравнения, така че да се получат технологични параметри за излужването е необходимо да се използват числови методи. Този доклад представя математически модел на слоест изотермичен поток от постоянна двукомпонентна течна среда вътре в каверната по време на излужващия процес. Математическият модел и свързаният компютърен алгоритъм е поддържан от компютърна програма, която е способна да изчисли основните параметри на технологичния процес.

Mathematical modeling of leaching process

Basic assumptions

The laminar isothermal flow of incompressible two-component liquid (water and salt) inside the cavern during leaching process is considered taking into account such effects as free convection and convective diffusion.

Basic assumptions of the model discussed are as follows:

- the shape of cavern at the initial moment is cylindrical;
- dissolution of salt is occurring on the lateral surface only;
- leaching process is isothermal;
- flow of brine is laminar;
- leaching process is treated as two-dimensional;
- brine is considered as ideal incompressible liquid.

Input data are divided into three groups:

- geological and physical parameters (densities of rock salt, water and saturated brine);
- geometric parameters (height and radius of cavern, radii of hanging coaxial pipes);
- Technological parameters (method of water injection – direct or reverse, rate of water injection, configuration of hanging pipes).

Main equations (mathematical model)

Flow of brine inside the cavern while leaching process may be described by following equations (in terms of dimensionless variables) (Кочин *et al*, 1963):

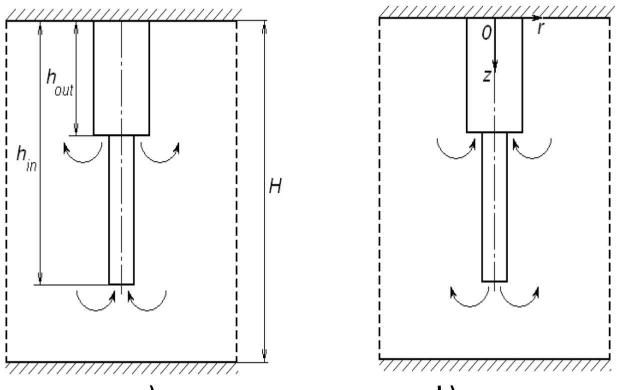

Fig. 1. Two basic methods of water circulation: a) – reverse, b) – direct.

Output data are:

- field of flow velocities;
- distribution of brine concentration within cavern;
- concentration of produced brine;
- rate of brine withdrawal;
- rate of rock salt dissolution.

- continuity equation: $\frac{1}{r}\frac{\partial (ru)}{\partial r} + \frac{\partial v}{\partial z} = 0$; (1)
- Navier-Stokes equations for each components of velocity:

$$\rho\left(\frac{\partial u}{\partial t}+\frac{1}{r}\frac{\partial\left(ru^2\right)}{\partial r}+\frac{\partial\left(vu\right)}{\partial z}\right)+\frac{\partial P}{\partial r}=\frac{1}{r}\frac{\partial}{\partial r}\left(\mu r\frac{\partial u}{\partial r}\right)+\frac{\partial}{\partial z}\left(\mu\frac{\partial u}{\partial z}\right); \quad (2)$$

$$\rho\left(\frac{\partial v}{\partial t}+\frac{1}{r}\frac{\partial\left(ruv\right)}{\partial r}+\frac{\partial v^2}{\partial z}\right)+\frac{\partial P}{\partial z}=\rho\cdot Gr+\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial v}{\partial r}\right)+\frac{\partial}{\partial z}\left(\mu\frac{\partial v}{\partial z}\right); \quad (3)$$

- diffusion-convection equation:

$$\frac{\partial a}{\partial t}+\frac{1}{r}\frac{\partial\left(rua\right)}{\partial r}+\frac{\partial\left(va\right)}{\partial z}=\frac{1}{Sh}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial c}{\partial r}\right)+\frac{\partial}{\partial z}\left(\mu\frac{\partial c}{\partial z}\right)\right]$$

; (4)

- equation of brine condition:

$$\rho = 1+A\cdot a;$$

(5)

where u, v – components of velocity in cylindrical coordinates (r,z); ρ - density of brine; P - pressure; a - salt concentration in brine; $c=\frac{a}{\rho}$ - specific concentration of salt; μ - dynamic viscosity of brine; g - free fall acceleration; D - diffusion coefficient, Gr и Sh - Grashof and Schmidt numbers respectively.
The process is considered within the area shown in Fig. 2.

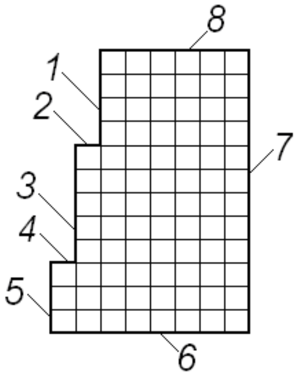

Fig. 2. Geometry of computational area: 1, 3 – lateral surfaces of hanging pipes; 6 – bottom of cavern; 8 – ceiling of cavern; 2, 4 – cross-sections of water injection and brine production; 5 –symmetry axis; 7 – surface of dissolution.

Initial conditions

It is assumed that at the initial moment ($t=0$) cavern is filled by stationary saturated brine: $u=v=0$. Initial concentration of salt, brine density and pressure are expressed as follows:

$$a=A_n,\ \rho=1+A\cdot A_n,$$
$$P=Gr\cdot\left(1+A\cdot A_n\right)\cdot\left(z-z_{in}\right),$$

where z_{in} - dimensionless height (z-coordinate) of water injection level, A_n - salt concentration in saturated brine.

Boundary conditions

The boundary of computational area consists of eight parts (Fig. 2) with different conditions on them:

- lateral surface of main hanging pipe (boundary 1) $r=r_{out}$; $0\le z\le h_{out}$:

$$u=v=0,\ \frac{\partial a}{\partial r}=0;$$

- boundaries of hanging pipes cross-section at the water injection level (boundary 2 or 4):

$$u=0,\ v=v_{in}=\frac{S_k}{S_{in}}\mathrm{Re},\ a=0,\ P=P_{in},$$

- boundaries of hanging pipes cross-section at the level of brine production withdrawal (2 or 4): $u=0$, $v=v_{out}$,

where: $\mathrm{Re}=\frac{R\rho_0 Q}{\mu\cdot S_k}$ - Reynolds number, $S_k=\pi R^2$,

$$S_{in}=\begin{cases}\pi\cdot\left(r_{out}^2-r_{in}^2\right), & 0\le r\le r_{in};\ z=z_{in};\\ \pi\cdot r_{in}^2, & r_{in}\le r\le r_{out};\ z=z_{out};\end{cases}$$ -

depending on reverse or direct circulation;

- lateral surface of central hanging pipe (boundary 3) $r=r_{in}$; $h_{out}\le z\le h_{in}$: $u=v=0$, $\frac{\partial a}{\partial r}=0$.
- symmetry axis (boundary 5) $r=0$; $h_{in}\le z\le H$:

$$u=0,\ \frac{\partial v}{\partial r}=\frac{\partial P}{\partial r}=\frac{\partial a}{\partial r}=0;$$

- bottom of cavern (boundary 6) $0\le r\le R$; $z=H$:

$$u=v=0,\ \frac{\partial a}{\partial z}=0.$$

- surface of dissolution (boundary 7) $r=R$; $0\le z\le H$:

$$a=A_n,\ v=0,\ u=\beta\cdot w,$$
$$\beta=1-\frac{\left(\rho_0+A_n-\rho_n\right)\rho_s}{A_n\cdot\rho_0},$$
$$w=\frac{\rho_0\rho_n}{\rho_s\left(\rho_n-A_n\right)}\frac{1}{Sh}\frac{\partial c}{\partial r}.$$

- ceiling of cavern (boundary 8) $r_{out}\le r\le R$; $z=0$:

$$u=v=0,\ \frac{\partial a}{\partial z}=0.$$

Numerical method

As it was told above the equations presented are to be solved numerically (Амосов *et al, 1994*). For this sake a computational three-dimensional mesh is needed (time and two space coordinates). Equations are approximated by means of explicit first order finite-difference scheme. The system obtained is solving by iterative methods. At the beginning field of pressure is calculated from continuity equation. This solution can be found analytically. Field of pressure determines field of velocities. Field of concentration may be found as a final stage.

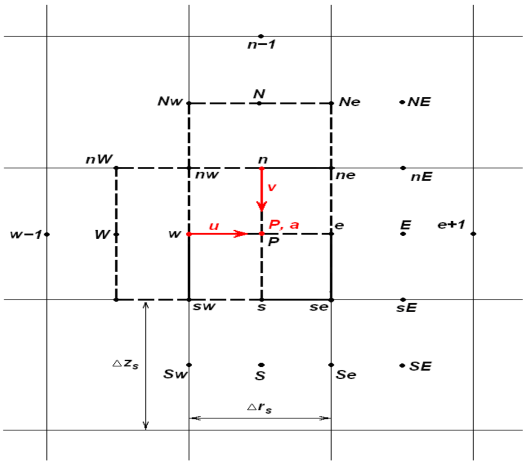

Fig.3. Template of finite-difference scheme: (nw-ne-se-sw) – computational mesh for pressure and concentration; (s-n-nE-sE) – for r-component of velocity; (w-Sw-Se-e) – for z-component of velocity.

Computer program

The enounced mathematical model and corresponding numerical algorithm are realized as a computer program for Windows. The prime objective of this program is to predict main characteristics for next stage of leaching process. For this purpose fields of velocities, pressure or concentration are calculated (see Fig. 4 - 7). The program enables to determine such parameters as concentration of produced brine, time of leaching as well as volume and shape of cavern. So, this program may be used as a practical tool to simulate basic technological process of salt cavern leaching and thus to predict shape and volume of prospective cavern under conditions and input parameters. Through multiple computations one can choose optimal decisions under different criteria, say, to minimize energy consumption or time of leaching.

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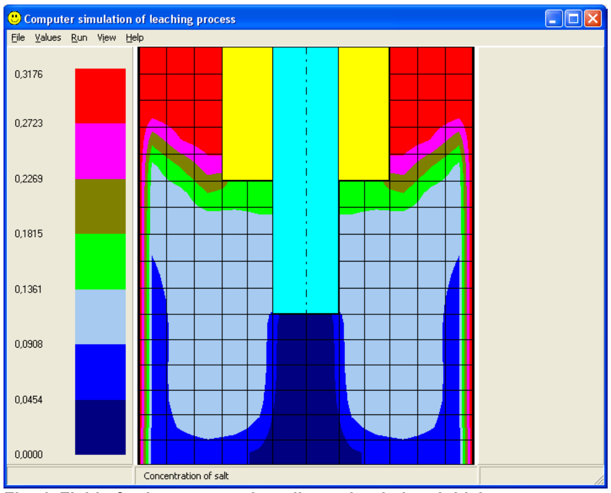

Fig. 4. Field of salt concentration, direct circulation, initial stage

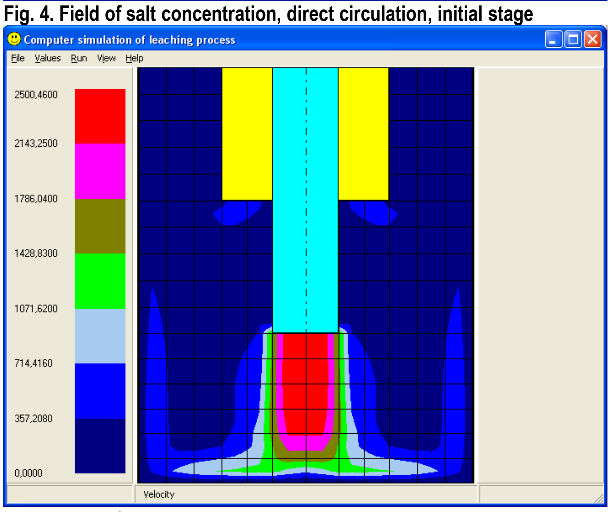

Fig. 5. Field of brine velocities, direct circulation, initial stage

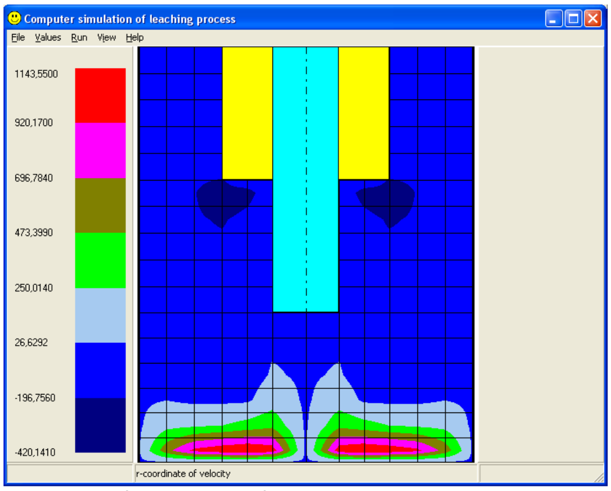

Fig. 6. Field of r-component of brine velocities, direct circulation, initial stage

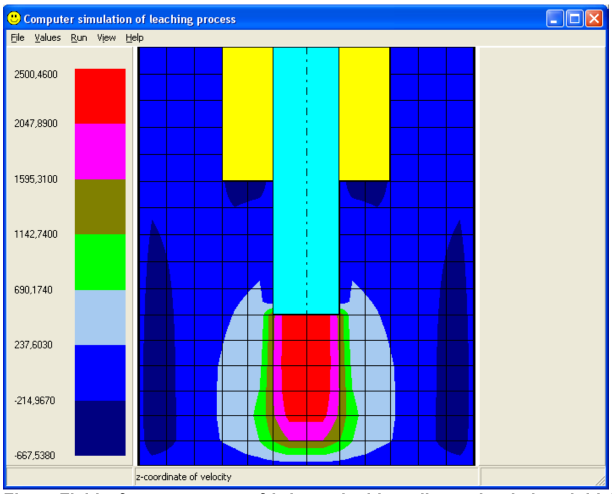

Fig. 7 Field of z-component of brine velocities, direct circulation, initial stage

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