Минно-геоложки университет "Св. Иван Рилски" Годишник, том 47, свитък I, Геология и геофизика, София, 2004, стр. 267-274

# SOME PRELIMINARY RESULTS OF THE ANALYSIS OF THE ALCALA DE RBRO VILLAGE (Zaragoza, Spain) GRAVITY ANOMALY WITH A SET OF POINT SOURCES

# Z. Zhelev <sup>1</sup>, T. Petrova <sup>2</sup>, F. Montesinos <sup>3</sup>, R. Vieira <sup>4</sup>, A. Camacho <sup>5</sup>

1.2 Geophysical Institute, Bulgarian Academy of Sciences (BAS), 1113 Sofia, e-mail:jelev@geophys.bas.bg, Petrova@geophys.bas.bg
3.4.5 Instituto de Astronomia y Geodesia, (CSIC-UCM), Ciudad Universitaria, 28040 Madrid, Spain, e-mail: fuen@mat.ucm.es, vieira@mat.ucm.es, Camacho@eucmax.sim.ucm.es

ABSTRACT. Several terrain's collapses took place in the last years inside Alcala de Ebro village (Zaragoza, Spain). Ebro river is close to the village and acts on this zone in an active way. The existence of cavities filled with water or sediments is supposed. The depth of these cavities may be around 12, 20 m. Besides, there are no other geological studies in the area that provide more information. That is why, we tried to use the available geophysical information for this purpose. The Alcala de Ebro (Zaragoza) gravity anomaly was studied with a set of point masses model. After a preliminary polynomial approximation to eliminate the main part of the regional trend (so that to ease the optimisation proccess) the local gravity anomalies together with the rest of the trend are modeled with a set of elementary point sources and a linear trend. The unknown parameters of the suggested model are determined through optimisation. The obtained results seem to be quite in agreement with the carstic cavities filled with water or sediments supposed to be at different depths and the terrain collapses that have taken place in the last years in this region (Alcala de Ebro village - Zaragoza, Spain).

# НЯКОИ ПРЕДВАРИТЕЛНИ РЕЗУЛТАТИ ОТ АНАЛИЗА НА ГРАВИТАЦИОННАТА АНАМАЛИЯ ОКОЛО СЕЛО АЛКАЛА ДЕ ЕБРО (Сарагоса, Испания) СЪС СИСТЕМА ОТ ТОЧКОВИ МАСИ

# Ж. Желев <sup>1</sup>, Т. Петрова <sup>2</sup>, Ф. Монтесинос <sup>3</sup>, Р. Виера <sup>4</sup>, А. Камачо <sup>5</sup>

<sup>1,2</sup> Геофизичен Институт, Българска Академия на Науките (БАН), 1113 София, e-mail: jelev@geophys.bas.bg, petrova@geophys.bas.bg
<sup>3,4,5</sup> Институт по астрономия и геодезия, Университет Клудат, 28040 Мадрид, Испания, e-mail: fuen@mat.ucm.es, vieira@mat.ucm.es, Camacho@eucmax.sim.ucm.es

РЕЗЮМЕ. През последните години, в района на село Алкала де Ебро (Сарагоса, Испания) стават някои пропадания на терена. Река Ебро е близо до селото и оказва едно активно въздействие върху този процес. Предполага се съществуването на пещери пълни с вода или седименти. Дълбочината на тези карстови образования е вероятно от порядъка на 12 - 20 м. Освен това, няма други геоложки изследвания в района, които да дадат допълнителна информация. Именно затова, в случая се прави опит да се използва наличната геофизична информация за постигане на тази цел. Локалната гравитационна аномалия Алкала де Ебро (Сарагоса) се изследва със система от подвижни точкови маси. След предварително полиномоално апроксимиране за елиминиране на главната част на регионалния тренд (така че да се олесни оптимизационния процес) локалната гравитационна аномалия заедно с остатъка от тренда се моделират със система от елементарни точкови източници и линеен тренд. Неизвестните параметри на предложения модел са определени чрез оптимизация. Получените резултати изглеждат напълно в съгласие с катсровите образования изпълнени с вода или седименти, залягащи на предполагаема дълбочина от около 12 - 20 м и съорветните пропадания на терена ставащи през последните години в този район (село Алкала де Ебро - Сарагоса, Испания).

#### Introduction

Several terrain collapses took place in the last years inside Alcala de Ebro village (Zaragoza, Spain). Ebro river is close to the village and acts on this zone in an active way. The existence of cavities filled with water or sediments is supposed. The depth of these cavities may be around 12, 20 m. Besides, there are no other geological studies in the area that provide more information. That is why, the geophysical observations are the only way to get some new ideas about the underground structure of this site. Specially here, we tried to use the available gravity data for this purpose. Of course, similar investigation in this region has been already made (Camacho et al., 1995) and interesting results have been obtained.

Besides, different gravimetric methods have been also employed for studying the origin, structure or activity of some other areas of this kind, and useful results have been obtained (Camacho et al., 1991; 1992; 1997; 2000 2001; 2002). In this sense, it may be said, that gravity modelling plays an important role in studying similar structures.

The aim of the present work is to contrast and to try to improve further the interpretation of the Zaragoza gravity data, using the above mentioned method of the solution of the inverse problems with a set of mobile elementary sources (ES) (Zidarov, 1965, 1968, 1990), which is known as very effective in similar situations (Zidarov, 1965, 1968, 1990; Bochev et al. 1974; Zidarov et al. 1970; Zhelev 1970, 1972, 1974, 1985, 1991, 1992, 1994; Zhelev et al. 1985, 1994). Besides, owing to

the well known ambiguity and instability of the solution of these problems, the application of different methods for their solution is very encouraging and perspective, even necessary, for the achievement of more real results.

## Gravimetric and topographic data

On the basis of some suitable topographic and gravimetric observations, the corresponding maps are prepared. Fig. 1 shows the resulting terrain model. The respective refined Bouguer anomaly is given on Fig. 2 .

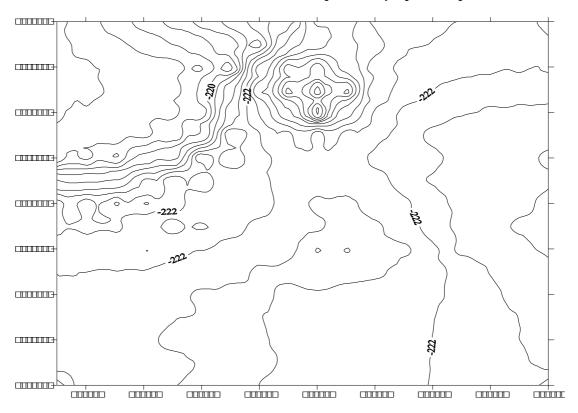


Fig. 1. Topographic model of the studied region. Contour interval 0.4 m, co-ordinates in meters

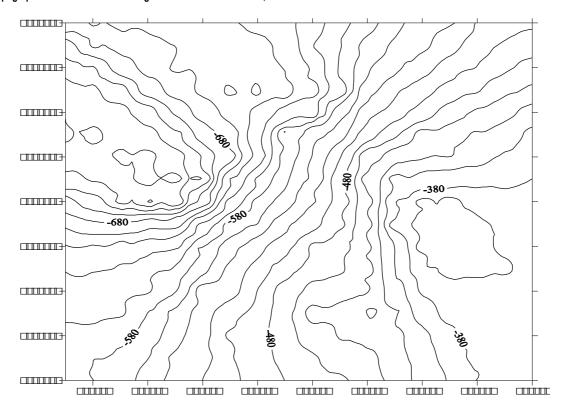


Fig. 2. Refined Bouger anomaly observed. Contour interval 20 mGal, coordinates in meters

### Mathematical formulation of the problem

The solution of the given problem by the above mentioned method (Zidarov, 1965, 1968, 1990) reduces mainly to the solution of the following non linear system of equations f(x) = y, where  $X(x_1, x_2, ..., x_n)$  is the vector of the unknown parameters (co-ordinates -  $\xi_k$ ,  $\eta_k$ ,  $\zeta_k$  and masses  $m_k$ , k = 1, ..., n/4 of the point sources (PS)), which must be determined

on the basis of the vector of the observations  $y^*$  ( $y_1, y_2, \ldots, y_N$ ), while  $f^*$  ( $f_1, f_2, \ldots, f_N$ ) is a vector of non linear functions (the symbol \* means transposition). In this case the functions  $f_i$ ,  $i=1,\ldots,N$ , can be defined by the analytical expression for the gravity effect ( $y=66.7 \cdot 10^{-12} \, m^3 \, kg^{-1} \, s^{-2}$  is the gravity constant) of a set of n/4 ES in N points of observation with coordinates -  $X_i$ ,  $Y_i$ ,  $Z_i$ ,  $i=1,\ldots,N$ , located over the Earth's surface.

$$f_i(x) = \sum_{k=1}^{n/2} \frac{\gamma \, m_k (z_i - \zeta_k)}{R_{ik}^{\square/\square}}, \quad R_{ik}^{\square} = (X_i - \zeta_k)^{\square} + (Y_i - \eta_k)^{\square} + (Z_i - \zeta_k)^{\square},$$

For the representation of the trend of the field, when necessary, a part a + bX + cY + ... (X and Y are the coordinates of the observational points) of a polynomial is used, whose coefficients - a, b, c, ... are determined in the process of optimisation, together with the rest of the unknowns (Zhelev, 1991, 1994). Alternatively, more PS can be included in the model for this purpose (Zhelev, 1991, 1994). Their parameters can be specified in the same way. Usually, in order to represent the trend, they must lie significantly deeper than the rest. Of course, the best thing to do here is to try to remove the trend before further interpretation, but this is not always possible with the needed precision. Even so it must be tried, because in all cases, this can considerably ease the optimisation in the next step (Zhelev, 1991, 1994).

#### **Numerical results**

As was already mentioned, the above described gravity anomaly was treated with this method (Zidarov, 1965, 1968, 1990). An appropriate Computer Program on FORTRAN 77, worked out by Zh. Zhelev (1970, 1972, 1974, 1991, 1994), was applied for this purpose. The Bouguer gravity was used obtained on the basis of the surface registrations. The observed anomaly and the corresponding trend were represented with 10 PS and the linear part of a polynomial. The optimisation was carried out after the Marquardt (1963) method.

A part of the results obtained - the parameters of the elementary sources - x, y and z co-ordinates and the masses, respectively -  $\xi_k$ ,  $\eta_k$ ,  $\zeta_k$ ,  $m_k$  and some quantities connected with the corresponding errors in the solution are presented in the Table. Besides, the following parameters are listed there for convenience:

- the functional

$$F(x) = \sum_{i=1}^{N} [y_i - f_i(x)]^{\square},$$

- the corresponding mean square deviation (MSD)

$$\sigma_{V} = [F(x)/(N-n-m)]^{\square \square},$$

- the gradient of the functional

$$G(x) = \left\{ \sum_{k=0}^{n} \left[ \frac{\partial F(x)}{\partial x_k} \right]^{-1} \right\}^{-1},$$

- the coefficient of non-representativeness (Zhelev, 1970, 1972, 1974)

$$K_{v} = \Box \Box \left( F_{E} \right) \Box \Box \qquad o_{O} , \quad E = \sum_{i} y_{i}^{\Box} ,$$

at the point of the minimum, etc.

Besides, some of the results are also presented on suitable illustrations - figures 3, 4 and 5.

On Fig. 3, the field of the obtained model is given.

On Figs. 4 and 5, the residuals (the difference between the observations and the models field) and the obtained solution are represented, respectively. The locations of the point sources of the solution are shown with small circles (the number in them corresponds to the number placed in the k-th column of the table). Besides, the plan view of the above mentioned (Alcala de Ebro) village and river can be also seen on the last picture.

Table.

Solution of the Inverse Gravity Problem with a system of elementary sources – ten point sources and a linear trend represent the local field of the "Zaragoza" gravity anomaly (see Fig.3 and Fig. 4)

| ar 11010 01 1110 20                                     |   |                    |    |              |             |             |                      |
|---|---|--------------------|----|--------------|-------------|-------------|----------------------|
| F   | G   | K <sub>ν</sub> [%] | k  | $\xi_{k}[m]$ | $\eta_k[m]$ | $\eta_k[m]$ | $M_k[kg / 1.5.10^2]$ |
| .44 . 10 8  | .95 . 10 <sup>7</sup>   | 69.42              | 1  | 1144.00      | 1135.00     | -100.00     | 3142100.00           |
| Initial approximations:                                 |   |                    | 2  | 998.00       | 1247.00     | -113.00     | -712610.00           |
|   |   |                    | 3  | 1140.00      | 1190.00     | -47.00      | -760670.00           |
|   |   |                    | 4  | 975.00       | 1247.00     | -113.00     | 716560.00            |
|   |   |                    | 5  | 1018.00      | 1230.00     | -193.00     | 1076.00              |
|   |   |                    | 6  | 1143.00      | 1139.00     | -98.00      | -7058400.00          |
|   |   |                    | 7  | 999.00       | 1216.00     | -92.00      | -79908.00            |
|   |   |                    | 8  | 1142.00      | 1142.00     | -103.00     | 3252300.00           |
|   |   |                    | 9  | 1138.00      | 1170.00     | 53.00       | 1596100.00           |
|   |   |                    | 10 | 975.00       | 1150.00     | 185.00      | 6200.00              |
| Description of the travel (a.h.a.)                      |   |                    |    | -760.95      | 0.31235     | -0.11306    |                      |
| Parameters of the trend (a h s)                         |   |                    |    |              |             |             |                      |
| .31 . 10  | 5 .19 . 10  | 4 1.83             | 1  | 1121.88      | 1154.81     | -103.96     | 3481775.70           |
|   |   |                    | 2  | 1008.84      | 1241.82     | -160.47     | -841375.96           |
| Solution :  |   |                    | 3  | 1104.99      | 1193.27     | -3.27       | -950360.35           |
|   |   |                    | 4  | 1007.76      | 1242.57     | -160.06     | 749872.44            |
| N = 294 , n = 56 , m = 3 ;                              |   |                    | 5  | 1015.47      | 1236.61     | -167.90     | 78477.41             |
| $\sigma_{\nu} = 1.70 \text{ mgal}$ .                    |   |                    | 6  | 1118.65      | 1158.25     | 101.72      | 7925777.70           |
|   |   |                    | 7  | 826.67       | 1236.68     | -115.43     | -13964.10            |
|   |   |                    | 8  | 1116.28      | 1160.94     | -103.99     | 3603840.50           |
|   |   |                    | 9  | 1111.55      | 1169.22     | -55.71      | 1660897.50           |
|   |   |                    | 10 | 920.39       | 1166.57     | -162.02     | -6500.05             |
| Parameters of the trend (a.b.s):                        |   |                    | 11 | -915.45      | 0.44429     | -26.59      |                      |
| Corresponding confidence inter-                         |   |                    | 1  | 1.9021       | 1.8905      | 2.0947      | 10515.7964           |
|   | vals : $\delta \ x_k = \pm \sigma_k \ t_{(N\text{-}n\text{-}m),\alpha}$ , |                    | 2  | 2.5464       | 2.2446      | 2.8014      | 8742.8855            |
| $\sigma_k = \sigma_v W_k$ , $W_k = [a_{kk}]^{1/2}$ ;    |   |                    | 3  | 4.6226       | 5.2497      | 6.6286      | 10568.4704           |
| σ   | $\sigma_v = 1.70 \text{ mgal}$ ,  |                    | 4  | 2.5660       | 2.2631      | 2.8502      | 8743.4649            |
| $(t_{235,0.05} = 1.971)$                                |   |                    | 5  | 2.4302       | 2.1500      | 2.2149      | 0700.5650            |
|   | $t_{235,0.01} = 2.599$ ),   |                    | 6  | 1.9665       | 1.9318      | 1.4380      | 10517.2819           |
| a kk, k=1,,n+m are the diagonal elements of the inverse |   |                    | 7  | 7.0833       | 4.3218      | 10.4795     | 3010.1955            |
| of the matrix of the correspond-                        |   |                    | 8  | 2.1670       | 2.1333      | 2.2261      | 10515.5263           |
| ing normal system of equations at the                   |   |                    | 9  | 3.2066       | 3.5799      | 2.3627      | 10576.2924           |
| point of the minimum, arranged consequently by rows :   |   |                    | 10 | 2.3498       | 2.0457      | 4.7407      | 1048.7404            |
|   |   |                    |    | 32.6195      | .0224       | .0269       | .510.1101            |
| Parameters of the trend (a b s):                        |   |                    |    | 02.0100      | .0221       | .0200       |                      |

Obviously, to obtain the masses in kilograms, the given quantities in the table must be multiplied by 1.5  $10^2$ . For example: spheres with 0.360 gr/sm³ density (i.e. - one approximately real density contrast) and 5 m radius, have a mass about 1.884 .  $10^5$  kg, t.e. more or less of the same order, as the masses of the main PS of the obtained solution.

The real x and y co-ordinates -  $\xi_k$  and  $\underline{\eta}_k$  respectively (in meters) can be obtained on the base of the following relations:

$$\xi_k = \xi_k + \; \text{649 000 [m]} \;\; , \;\; \underline{\eta}_k \; = \; \eta_k + \; \text{4 629 000 [m]} \; . \label{eq:xi_k}$$

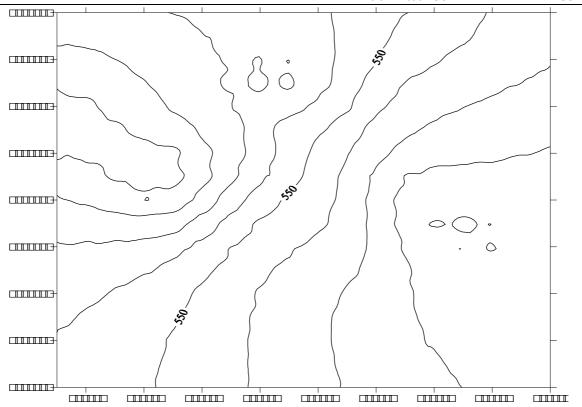


Fig.3. The model's .field. Contour interval 50 mGal, coordinates in meters

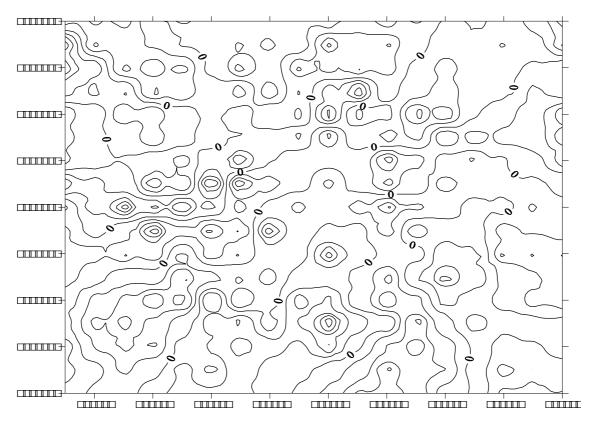


Fig. 4. The residuals. Contour interval 0.4 mGal, coordinates in meters

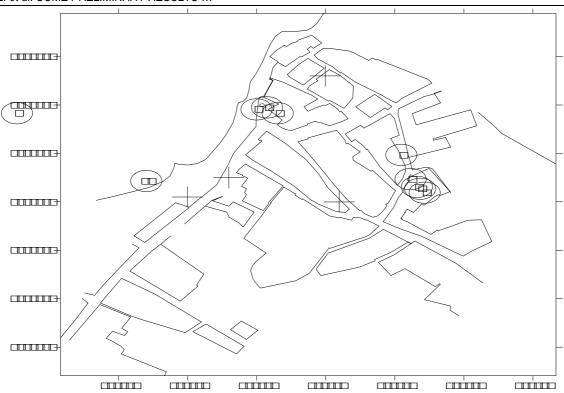


Fig. 5. The obtained solution - the locations of the respective point sources are shown with small circles (the number in them corresponds to the number placed in the k-th column of the table). Besides, the plan view of the above mentioned (Alcala de Ebro) village and river can be also seen on this picture

Taking into account that the polynomial of first degree, represents mainly the trend of the field in this region, the obtained PS confirm more or less the structural model, obtained by (Camacho et al., 1991, 2001) several years ago. As can be easily seen from the table and Fig. 3, the main anomalous masses (about 90 % of them), probably related with the main collapses in this region, are concentrated in tow groups in the north and north-east parts of the studied area, approximately at the depth - 30 - 60 m. One of the PS is in the west part of the region, approximately at a depth of about 60 m, probably connected with some other peculiarities of this location. However the present investigation gives also something new - here (on the basis of the obtained results see the Table and Fig. 5) it may be suggested, that probably some of the terrain collapses in this region are connected with deeper inclined (strongly shifted in depth) carstic cavities, filled with water or sediments.

On the figures, some other details are also given for convenience.

The values of the functional, the corresponding MSD and the coefficient of non-representativeness (see the Table) show that a satisfactory solution is found, and the model fits the observations comparatively well. The corresponding gradient points out that the optimum of the functional has been approximately achieved. Moreover, the coefficient of non-representativeness shows that part (in percentage) of the observations, which is not presented by the model used.

The corresponding MSD is about 10.81 *mgal*, i.e. it is approximately of the order of the observational error, which is more or less in agreement with the theory (Zhelev, 1991, 1994; Draper et al., 1986; Tihonov, 1965; Wiggins, 1972).

As a matter of fact, the obtained MSD for the whole observational region is a little larger than the corresponding

mean square observational error. This is mainly connected with the comparatively large residuals (systematic part) in the whole observational area (see Fig. 4), related mainly with some details, which are not entirely presented with this model. Obviously in the future some more complicated model must be used for this purpose, on the basis of course of some new, more detailed and precise observations, with an improved method.

#### Stability and errors in the solution

As is known (Draper et al., 1986), the problem concerning the exact evaluation of the errors in the solution in the non-linear case is not satisfactorily solved yet. But as in the close vicinity of the minimum, a linear representation is usually acceptable, the well-known formalism concerning statistical estimations of linear systems (Draper et al., 1986) can be used in this and similar cases to study the stability of the solution. An approximation of the corresponding confidence intervals  $\delta x_k$  of the unknowns can be obtained by the following expression (Draper et.al., 1986) where  $a_{kk}$ , k = 1, ..., n are the diagonal elements of the inverse matrix of the respective normal system of equations, and  $t_{(N-n-m),\alpha}$  is the corresponding t score for the respective degrees of freedom (N-n-m) (m - the number of the trend parameters) and level of certainty  $\alpha$ . Thus, we can have an approximate idea about the confidence intervals, suggesting an almost linear connection between the unknowns and the observations at the point of the minimum and its surroundings.

$$\delta x_k = \pm \sigma_k t_{(N-n-m),\alpha}, \sigma_k = \sigma_v \omega_k, \omega_k = [a_{kk}]^{\square \square},$$

As can be easily seen from the table, almost all the PS and the trend are comparatively well determined - the

corresponding errors in the solution are within acceptable limits

Naturally, when this method is used, the question how to determine the optimal number of model parameters is essential. Although the optimization method used automatically eliminates the extra parameters of the model, in order to ease the optimization process however, the following additional method (Zhelev, 1991, 1994) can be employed for this purpose.

The problem can be solved for different numbers of elementary sources. The number n at which the corresponding MSD has a minimum, had to be chosen as an optimal one. It is not difficult to show, that if the number of the observations is large enough, there is a number of the ES at which this criterion has a minimum and this optimum coincides with the real number of the parameters of the source - the respective proof can be seen in (Zhelev, 1991, 1994). Obviously, the minimum value of the MSD thus obtained, must be approximately equal (or a little less) to (than) the corresponding mean square error in the observations, as it is its unbiased estimate (if a good representation is achieved) (see Zhelev, 1991, 1994). Thus, instead of searching for the minimum, we can look for that n for which the corresponding MSD coincides with (or is a little less than) the respective mean square error in the observations, when it is known of course (Zhelev, 1991, 1994).

#### Conclusion

It may be said in conclusion, that some new results are obtained here, by a different (mathematically well-grounded) method, which confirm the structural model obtained in previous works and gives with greater certainty and precision a more detailed idea about the distribution of the anomalous masses in depth in this region. More specially, here it is suggested, that some of the terrain collapses are probably connected with some deeper carstic cavities, supposed to exist in this region. On the basis of all this and the proven in practice possibilities of the method used here (Zidarov, 1965, 1968, 1990; Bochev et al., 1974; Zidarov et al., 1970; Zhelev, 1970, 1972, 1974, 1985, 1991, 1992, 1994, 1996; Zhelev et al., 1985; 1996), we can hope that now we already have one more precise and real idea about the underground structure in this region.

It must be added at the end, that all these results are obtained only on the basis of gravity observations and topographic information - without including other geophysical data. In spite of all this, the obtained results seem to be quite in agreement with the preliminary suggestions on the basis of some other information - carstic cavities filled with water or sediments supposed to be at different depths and the terrain collapses that have taken place in the last years in this region (Alcala de Ebro village - Zaragoza, Spain). Indeed, better results may be expected on the basis of some new, more detailed and precise observations, by an updated method. Of course, additional improvements can be expected also, including some magnetic observations in this study.

#### References

- Àrnoso, J., 1996. Modelizacion y evaluacion de efectos indirectos sobre las mareas terrestres en elarea de las Islas Canarias. *Ph. D. Thesis, Univ. Complutense* of *Madrid, Spain.*
- Bochev, A. and H. Georgiev, 1974. Solution of Inverse Gravity Problem in the Case of Point Sources by Using Least Squares and Linear Restrictions. *Rev. Ital. Geoph.* 23, 3/4, 167-172.
- Camacho, A. G., R. D. Vieira, 1991. Gravimetric Study of Lanzarote Island., *Tire a part des Cahiers du Centre Europeen de Geodynamique et de Seismologie*, **4**, p. 339 351
- Camacho, A.G., R. V. Vieira, F. G. Montesinos, J. Arnoso, 1992. Investigaciones gravimetricas sobre la estructura de la corteza volcanica en la isla de Lanzarote. in: International conference on Cartography-Geodesy, Maracaibo (Venezuela) II, pp. 364-372. eds: Sevilla, M.J., Henneberg, H. & Vieira, R, I.A.G. (CSIC-UCM) Madrid.
- Camacho, A.G., R. D. Vieira, F. G. Montesinos, y V. CuellarJ., 1994. A gravimetric 3D inversion for cavity detection, *Geophysical Prospecting* 42, 113-130.
- Camacho, A.G., F. G. Montesinos, R. D. Vieira, J. L. Valbiena and A. Alcazar, 1995. Estudio gravimetrico de anomalias del subsuelo en el area urbana de Alcala de Ebro (Zaragoza), *Instituto de Astronomia y Geodesia*, C.S.I.C. -U.C.M., Madrid.
- Camacho, A.G., F. G. Montesinos, & R. D. Vieira, 1997. A three-dimentional gravity inversion applied to S. Miguel island (Azores). *J.geophys. Res.*, **102**, 7717-7730.
- Camacho, A.G., F. G. Montesinos, R. D. Vieira, 2000. Graviy inversion by means of growing bodies. *Geohysics*, 65, 95-101
- Camacho, A.G., F. G. Montesinos, R. D. Vieira, and J. Arnoso, 2001. Modelling of Crustal Anomalies of Lanzarote (Canary Islands) in Light of Gravity Data, *Geophysical Journal International*, 147, 2, 403 - 414.
- Camacho, A.G., F. G. Montesinos, R. D. Vieira, 2002. A 3-D gravity inversion tool based on exploration of model possibilities. Computer & Geosciences, vol. 28, 2, 191-204.
- Draper, N. R. and H. Smith, 1986. Applied Regression Analysis. John Wiley and Sons, Inc. New York, London, Sydney.
- Marquardt, D. W., 1963. An Algorithm for Least-Squares Estimation of Nonlinear Parameters. *J. Soc. Ind. App. Math.*, **12**, 431-441.
- Zidarov, D. P., 1965. Solution of Some Inverse Problems of Applied Geophysics. *Geophysical Prospecting*, **13**, 2, 240 246
- Zidarov, D. P., 1968. On the Solution of Some Inverse Problems of the Potential Fields and its Application in the Exploration Geophysics. B A S. Sofia.
- Zidarov, D. P., 1990. Inverse Gravimetric Problem in Geoprospecting and Geodesy. Elsevier, Amsterdam -Oxford - New York - Tokyo.
- Zidarov, D. P. and Zh. P. Zhelev, 1970. On Obtaining a Family of Bodies with Identical External Fields Method of Bugling. *Geoph. Prop.* **18**, *1*, 13 33.

- Zhelev, Zh. P., 1970. Application of the Poincare's Method of Balayllage for Solution of the Inverse Potential Field Problems. M. Sc. thesis, Geophys. In st. of the BAS, Sofia.
- Zhelev, Zh. P., 1972. A Numerical Solution of the Inverse Gravimetric Problem with Application of the Method of Partial Balayllage. *Inv. Geoph. Inst.*, **18**, 143 156.
- Zhelev, Zh. P., 1974. Some Numerical Results of the Inverse Gravity Problem Solution with a Finite Number of Elementary Sources. *Compt. rend. Acad. bulg. Sci.* 27, 6, 771 794.
- Zhelev, Zh., H.M. Georgiev, 1985. Some Preliminary Results of the Representation of the Earth's Gravity Field as a Model of Elementary Sources. *Compt. rend. Acad. bulg. Sci.*, **38**, 3, 315-317.
- Zhelev, Zh. P., 1991. Some Results from the Representation of the World Gravity Anomalies with Elementary Sources.
  Proceedings of the 9-th Mathematical Geophysics Int.
  Seminar on Model Optimization in Geophysics, Vol. 5, (Ed. A. Vogel), Vieweg Braunschweig Wiesbaden, Berlin, p. 151 170.
- Zhelev, Zh. P., 1992. On the Precision of the Method of Grids and The Partial Balayllage (Sweeping out), *Bulgarian Geophysical Journal*, v. **18**, 3, p. 43 51.

- Zhelev, Zh. P., 1994. Representation of the World Gravity Anomalies Sources with a Final Number of Point Masses. *Bulgarian Geophysical Journal*, **20**, 2, p. 57 67.
- Zhelev, Zh. P., T. Petrova, R. Vieira, A. Camacho, F. Gonzalez, 1996. Some Results from the Interpretation of the Lozoyuela Gravity Anomaly with Elementary Sources, *Bulgarian Geophysical Journal*, **22**, 3, p. 51 62.
- Sevilla, M., R. Parra, 1975. Levantamiento gravimetrico de Lanzarote. *Rev. R. Ac. CC. Exactas, Fisicas y Naturales,* **69**, p. 257-284.
- Tihonov, A. N., 1965. On the Stability of the Algorithms for the Solution of the Degenerated Linear Systems of Algebraic Equations. *J. of Num. Math. and Math. physics*, **5**, 718.
- Vieira, R., M. Van Ruymbeke, J. Fernandez, J. Arnoso, C. Toro, C., 1991. The Lanzarote underground Laboratory. Cahiers du Centre Europeen de Geodynamique et de Seismologie. 4, 71-86.
- Wiggins, R., 1972. The General Linear Inverse Problem: Implication of Surface Waves and Free Oscillations on the Earth Structure. *Rev. Geophys. Spac. Phys.*, **10**, 251-285.

Препоръчана за публикуване от катедра "Приложна геофизика", ГПФ