MATHEMATICAL MODELING AND MODAL ANALYSIS OF THE TWO DRUM DRIVER SYSTEM FOR BELT CONVEYORS

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ABSTRACT. In this work are theoretically analyzed the processes witch can produce vibration phenomena in forming process of the traction force between the drum driver and belt into the two drum driven belt conveyors. There are defined the natural frequencies and conclusions about potentiality resonances into the driver system in different working regimes.

МАТЕМАТИЧЕН МОДЕЛ И МОДАЛЕН АНАЛИЗ НА ДВУБАРАБАННА ЗАДВИЖВАЩА СИСТЕМА ЗА ЛЕНТОВИ ТРАНСПОРТЬОРИ

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РЕЗЮМЕ. Извършен е теоретичен анализ на процесите, които пораждат колебателни явления при формиране на теглителна сила между задвижващите барабани и лентовото платно в двубарабанните задвижващи системи на лентовите транспортьори. Определени са собствените честоти и са направени изводи относно възможностите за резонанс в системата на задвижване при различни работни режими.

1. Preface

As it is known the rubber - belt conveyor transport systems with two – drum driver stations are one of the most powerful and high – productivity machines with continuous production in the mining industry. Working together with mining machines such like bucket – wheel excavators gave the possibility to realize high – productivity mining technologies in open coal mining. That is why the work on improvement of their exploitation - technical parameters and power consumption decreasing is high significant.

2. Problem status

Fig. 1 shows the two – driver leading group for one of the leading drums working together with steel rope armored rubber – belt type St 3150 in RBT(**R**ubber – **B**elt **T**ransporter) 2250, which are expotated in open coal mines Mini Maritca Iztok EAD. This construction characterizes by the console pendulum mountings of the driver – transmissions groups (2X1000 kWt) on the both ends of shaft – 1. From previous FEM analyses of the deformations and stresses under staticial loading of this type shaft (Дамянов 2007) is known that it is subjected to the significant cyclic bending deformations with cycle frequency "f_b" about *1 Hz*.

The other side of the problem is that often in exploitation are leading stations with some misalignment in mounting of the gear reducers to the leading drum shaft. Unsuitable combination of these constructive – technically factors is the reason to provoking the cyclic loadings in all of the working regimes of the conveyor elements.

The aim of this research is to establish the risks of inducing resonant regimes in belt conveyor leading system.

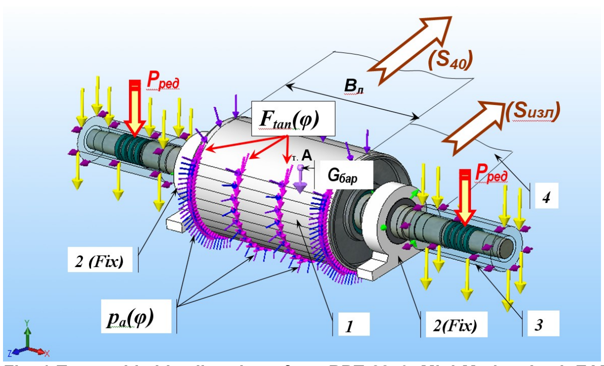

Fig. 1 Two – sided leading drum from RBT 2250, Mini Maritca Iztok EAD -- 1-leading drum; 2- bearing supports; 3-pendulum – mounted gear reducers; 4 – belt

3. Modeling and simulation of the two – drum belt conveyor leading station.

This type of the researched object is chosen through its wide spreading and importance in two – drum leaded rubber belt conveyors, uppermost used in Mini Maritca Iztok EAD transporting systems. The exploitation expenses relative to extracted and transported material unit in these systems are significantly great than these for the bucket – wheel excavators. That's why the improvement in exploitation security of the RBT is with substantial importance. One way to achieve this is decreasing value of mechanical loadings in the system elements, subjected to the high statical and also to the high dynamical loads. Part of these loading is taken by the vibration loading and circumstances to induce resonant effects leading to unallowable high values of stresses in the belt conveyor components.

Main problem of this research is frequency analysis of modeled two–drum mechanical leading system shown in fig.2

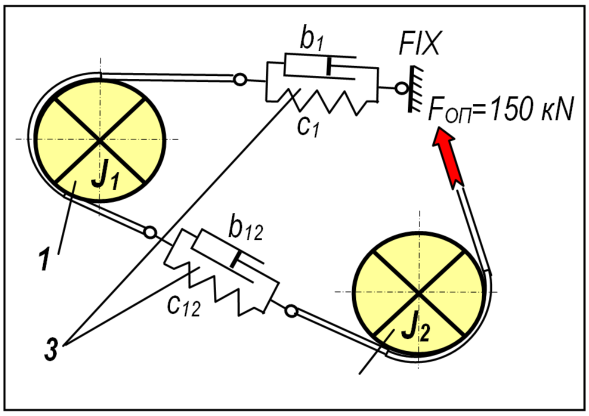

Fig. 2 Dynamical model of two – drum leading system with two degrees of freedom, where:
1 – first leading drum;
2 – second leading drum;
3 – elastic – damper units.

There will be considered two typically realizable starting cases for this system

3.1. *Two mass model with two degrees of freedom*

Shown on fig. 2 the reduced two – mass model represents mechanical system containing rotational moving mass elements with point masses and referred to leading drums (1 and 2) axles inertial moments (J_1 and J_2) connected through elastic – damper connection (3).

This model characteristics cases which starting of the system is by the simultaneously switching both of the leading drum groups. The model loadings contain both of leading groups load. In this case:

- Kinetic energy of the system is:

$$T = \frac{1}{2}\left[J_1.\dot{\varphi}_1^2 + J_2.\dot{\varphi}_2^2 \right]; \qquad (1)$$

- Potential energy of the system is:

$$\Pi = \frac{1}{2}\left[c_1.(\varphi_1.R_1)^2 + c_{12}.\left(\varphi_1.R_1 - \varphi_2.R_2\right)^2 \right] \qquad (2)$$

- Dissipative energy of the system is:

$$B = \frac{1}{2}\left[b_1.\left(\dot{\varphi}_1.R_1\right)^2 + b_{12}.\left(\dot{\varphi}_1.R_1 - \dot{\varphi}_2.R_2\right)^2 \right]. \qquad (3)$$

Considering using LaGrange differential equations from following form:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial \Pi}{\partial q_i} + \frac{\partial B}{\partial \dot{q}_i} = Q_i \qquad (4)$$

Differential equations describing the system's dynamics are:

$$\left| \begin{array}{l} J_1.\ddot{\varphi}_1 + c_1.\varphi_1.R_1^2 + c_{12}.R_1.(\varphi_1.R_1 - \varphi_2.R_2) + \\ \quad + b_1.\dot{\varphi}_1.R_1^2 + b_{12}.R_1.(\dot{\varphi}_1.R_1 - \dot{\varphi}_2.R_2) = 0 \\ J_2.\ddot{\varphi}_2 - c_{12}.R_2.(\varphi_1.R_1 - \varphi_2.R_2) - b_{12}.R_2.(\dot{\varphi}_1.R_1 - \dot{\varphi}_2.R_2) = Fon.R_2 \end{array} \right. \qquad (5)$$

Following this logic it is easy to determine the elements of the stiffness matrix by potential energy twice differentiation:

$$C_{i,j} = \frac{\partial^2 \Pi}{\partial q_i . \partial q_j}, \qquad (6)$$

which leads to stiffness matrix:

$$C = \begin{vmatrix} R_1^2.(c_1 + c_{12}) & -c_{12}.R_1.R_2 \\ -c_{12}.R_1.R_2 & c_{12}.R_2^2 \end{vmatrix} \qquad (7)$$

Analogically it is possible to determine damper coefficients matrix:

$$B = \begin{vmatrix} R_1^2.(b_1 + b_{12}) & -b_{12}.R_1.R_2 \\ -b_{12}.R_1.R_2 & b_{12}.R_2^2 \end{vmatrix} \qquad (8)$$

and mass elements matrix:

$$M = \begin{vmatrix} J_1 & 0 \\ 0 & J_2 \end{vmatrix}, \qquad (9)$$

The values of mass inertial moments of the leading drums with corresponding to them reduced point masses of the belt and burden are: J_1 = *98953 [kg.m²]*; J_2 = *29356 [kg.m²]*

Stiffness of the belt depends on the length of the interested section. For transporter belts with steel rope armors similar to type St 3150 is accepted resulting stiffness to be equal to the steel wire stiffness and neglecting stiffness of the rubber material.

Parameters of RBT belt type *St 3150* are*:*

Number of the steel ropes – n =146;

Steel rope diameter– d = 8 mm;

Steel rope section area - $A = \pi.d^2/4 = 5{,}03^{-5}\ m^2$;

Elasticity modulus value for steel used in the armoring ropes: $E_M = 2.10^{11}\ N/m^2$;

Considering ropes section to be continuous the used elasticity modulus is need to be reduced in: – $E = 5\ E_M/8 = 1{,}25.10^{11}\ N/m^2$; (Спиваковский, 1982 Шахмейстер, 1978)

Length of the first section is - L_1 *=246m*, of the second is - L_{12} *=20m.*

In this case the length of the first section is 1/6 from the full length of burden carrying line (de'Silva 2001), and the length of the second section is equal to the full length between the two leading drums.

In these conditions the stiffness of the belt is calculated by the following:

$$c_1 = \frac{n.A.E}{L_1} = 3{,}729.10^6 \ \frac{N}{m}$$
$$c_{12} = \frac{n.A.E}{L_{12}} = 4{,}5867.10^7 \ \frac{N}{m} \qquad (10)$$

Damper coefficients b_1 and b_2 are accepted to be 0,5% from the c_1 and c_2 (according to de'Silva 2001) and are calculated to be:

$$b_1 = 1{,}8645.10^4 \ \frac{N.s}{m}$$
$$b_{12} = 2{,}2934.10^5 \ \frac{N.s}{m} \qquad (11)$$

Definining amplitude – phase matrix in the following view:

$$W = \left[C - \omega^2.M + i.\omega.B\right]^{-1} \qquad (12)$$

and using *Matlab* to calculate the natural frequencies for the system: $f_1 = 0{,}6850$ *[Hz] and* $f_2 = 5{,}7454$ *[Hz]*; also Matlab gave the view of the amplitude – frequency characteristic shown on fig. 4.

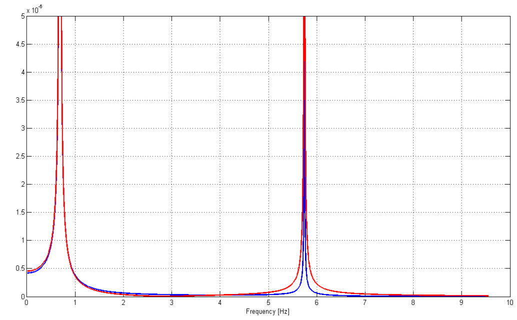

Fig. 3 Amplitude – frequency characteristic of two - drum driven system - $f_1 = 0{,}69$ ***u*** $f_2 = 5{,}74$

These results are confirmed with FFT spectrum analysis – shown on fig. 5. – of the elastic force – shown on fig. 4 – witch is produced between the leading drums when rising difference between their angular velocities(Дамянов, Лазов, Недялков 2007).

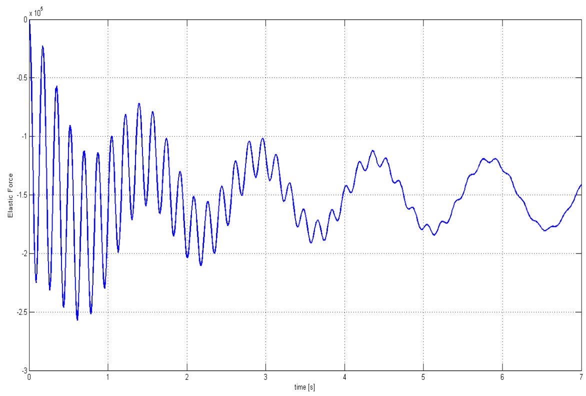

Fig. 4. Diagram of elastic force between the leading drums using two – mass dynamical model.

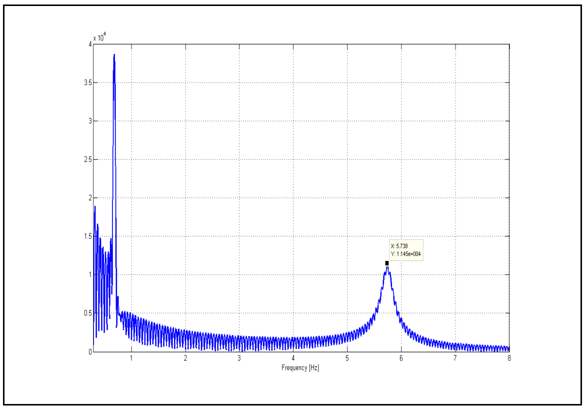

Fig. 5. Amplitude spectrum of the elastic force between the leading drums

3.2. Dynamical model with single degree of freedom

This part of the research deals with dynamical model of the same double drum driving station but when is switched on brake of the first driving drum. This case represent the real situation for the switching process with overtaken (2 seconds) switching of the second drum before switching the first drum. Using this control function of the driving electrical engines ensures always the needed value of the tension force in the belt witch is precondition for forming sufficient tractive force for the starting process (Дамянов 2007).

The scheme of dynamical model with single degree of freedom is shown on fig. 6. Moving elements are – second leading drum and belt unit between first and second drum.

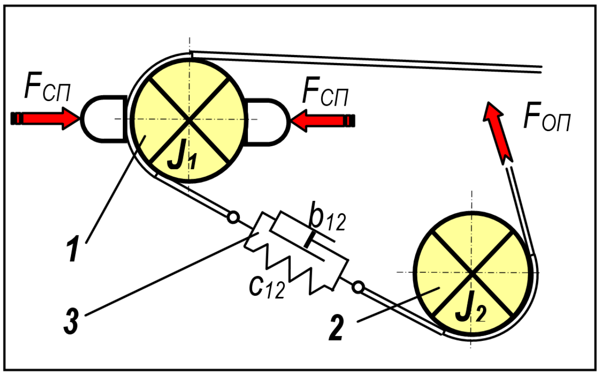

Fig. 6. Dynamical model of the double – drum driving system with single degree of freedom

Differential equation of motion is:

$$J_2.\ddot{\varphi}_2 - c.R_2^2.\varphi_2 - b.R_2^2.\dot{\varphi}_2 = Fon.R_2 \qquad (13)$$

$$\ddot{\varphi}_2 - \frac{c.R_2^2}{J_2}.\varphi_2 - \frac{b.R_2^2}{J_2}.\dot{\varphi}_2 = \frac{Fon.R_2}{J_2}$$

$$\ddot{\varphi}_2 - 2.n.\dot{\varphi}_2 - \omega^2.\varphi_2 = \frac{Fon.R_2}{J_2}$$

where:

$$\omega^2 = \frac{c.R_2^2}{J_2}; \ n = \frac{b.R_2^2}{2.J_2}; \qquad (14)$$

$\xi = \dfrac{n}{\omega}$ - relative damping;

J_2 = *29356 [kg.m²]* – reduced mass inertial moment with added belt mass.

Here is accepted greater value of relative damping factor ξ - 10%. Reasons to accept greater values for this factor are – using the maximal value of relative damping that will not change significant natural frequencies of the system(cause small influence over the natural oscillations of the system); - the value of this factor is not well known; - to permit experimental verification of model.

$$\omega_d = \sqrt{1 - \xi^2}.\omega = 0{,}99.\omega \qquad (15)$$

The values for stiffness of the belt are the same like two – degree of freedom model – *n, A и E* but with different value *for* L_2*=20 m*, result is – c_{12} = *4,5867.10⁷ [N/m]*. Drum radius is the same– R_2 *=0,8 [m]*.

Hence the natural angular frequency is – ω = *31,622 rad/s* and the cycle frequency is – $f = \omega / 2\pi$ = *5,0346 Hz*. In this case the frequency witch can cause resonant phenomena is only one.

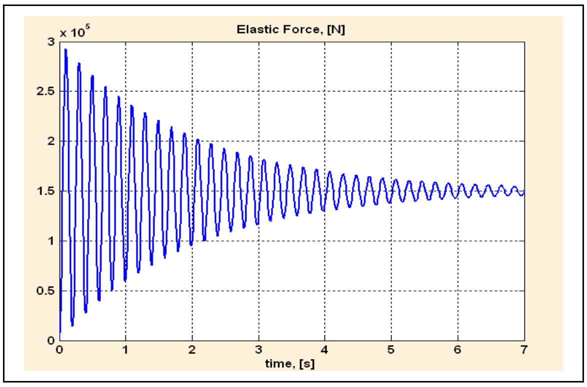

Fig. 7 Diagram of elastic force between the leading drums using single – degree of freedom model.

4. Conclusions

The upper results afford to draw general conclusions:

4.1 – Quantity appraisal of the results relates only for RBT 2250 from Mine 1 in Mini Maritca Iztok EAD and the quality appraisal relates for all RBT systems with double – drum driving and overtaken switching of the second drum.

4.2 – Model with single degree of freedom characterizes a brief starting part (2 to 3 seconds), in which oscillation processes envelop only second drum and belt unit with length 20 m – between two leading drums. In this case the circumstances for resonance are lacking because of high natural frequency – f_1 *=5.0346 [Hz]* and is greater than working frequency in steady state regime f_b*=1,17 [Hz]*

4.3 – In system represented with two – mass model there are preconditions for stimulation of resonance when leveling the first natural frequency f_{r1} *= 0,6850 [Hz]* and variable working frequency – f_b*=var (0 to 1,17 Hz)* ;

4.4 – Resonance phenomena which can cause linear oscillations are unacceptable because of the following reasons:

- Pulsing tension force – tension force in the belt start pulsing in the first drum outgoing section which decrease it's traction limit and increase preconditions for skidding;
- Additional pulsing loads – additional pulsing changes of the tension force in this section increase general loading acting at all mechanical elements of driver system – bearings, welding construction of the drum driver, drum shaft and etc.

Knowledge about and decreasing of vibrational loadings in powerful multi – driver leading stations of rubber – belt conveyors is important technical – exploitation problem. By this analysis and conclusions about existing possibilities for creation of low frequency resonance oscillations with high amplitude of tension force in the belt and all connected mechanical elements are shown the importance of researched problem and work about minimization of unfavorable effects about exploitation security and power consumption should follow the next generalities:

–Précising dynamical model according to constructive – technical parameters of the system for starting and stationary regime;

–Research about influence of tension force over frequency parameters of the system;

–Optimization of the control function of tension system;

–Organize the comparison experimental research and validation of the results;

References

1. Clarence W. de Silva, Vibration Fundamentals and Practice, CRC Press,Washington, 2001
2. Дамянов Ц., П. Недялков, Л. Лазов. 2007. Специални приложения на тензометрията за нискочестотна вибродиагностика в областта на минната техника. Год. МГУ „Св. Ив. Рилски". т 50. Свитък III. 51-54.
3. Спиваковский, А.О.и др.Теория ленточных конвейров, М. Наука., 1982
4. Шахмейстер, Л.Г. и др. Теория и расчет ленточных конвейров, М. Машиностроение, 1978
5. Дамянов, Цв. Л. и др. *Промишлени експериментални изследвания на лентовите транспортни системи (3х560 квт и широчина В = 1800 мм) в рудниците на "Мини Марица-изток" ЕАД* – Договор 1853/2004 при НИС на МГУ,Архив на НИС на МГУ., 2005
6. Дамянов Цв. Л. и др. *"Експериментално изследване и анализ на технико – експлоатационното състояние на ГТЛ 2250 №2 на рудник „Трояново -1" (чешка система) за условията на „Мини Марица Изток" ЕАД"*- Договор 1905/2007 при НИС на МГУ,Архив на НИС на МГУ., 2007
7. Дамянов Цв. Л. и др., „Тридименсионно, моделиране, изследване и анализ с метода на крайните елементи на якостно-деформационното състояние на задвижващите барабани от станциите на ГТЛ 2250 за условията на

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