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# APPLICATION OF ASSIGNMENT PROBLEM IN MINING

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**ABSTRACT:** The assignment problem belongs to the class of optimization problem. In this paper is given an example of usage of assignment problem solving for an process optimization in mining.

**ПРИЛАГАНЕ НА КАДРОВА ПОЛИТИКА В МИННАТА ПРОМИШЛЕНОСТ**

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**РЕЗЮМЕ:** Кадровата политика принадлежи към категория оптимизационни проблеми. В доклада е даден пример за решаване на кадрови проблем при оптимизиране процеса на минното производство.

## Introduction

One of the very important goals of project realization is its realization within the limits of the planned costs. When dealing with large projects, the right assignment of the workforce and of other resources can have a significant influence on the project total costs.

## The Model of Assignment

The main purpose of this model is the assignment of a certain number of workers or resources intended for specific jobs.

The optimization of the assignment of workers or resources in charge of specific jobs is realized through the model in which the function of the goal represents the demand for the minimization of costs of the given assignment, and can be represented as follows:

$$(\min)\ Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \qquad (1)$$

Whereby $x_{ij}$ stands for the variable which shows engagement or non-engagement of worker *i* in charge of job *j*, and $c_{ij}$ stands for the costs worker *i* makes during job *j*.

If worker *m* is supposed to be assigned job *n*, whereby the assignment is such that it requires minimal costs for doing the job, the assignment model will be the following:

$$(\min)\ Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$\sum_{i=1}^{m} x_{ij} = 1 \quad i = 1,2,...,n \qquad (2)$$

$$\sum_{j=1}^{n} x_{ij} = 1 \quad j = 1,2,...,n$$

The variable $x_{ij}$ can be either 0 or 1, and it shows engagement, that is non-engagement, of worker *i* for job *j.*

Thus defined model represents a special form of the linear programming model (the so-called 0-1 programming)

In order to solve the assignment problem, the number of workers has to be equal to the number of jobs (m= n).

The importance of solving the problem defined in this way lies in the fact that there is a large number of possible assignments, but optimization means that a worker is assigned a job which secures the lowest total costs.

**Mathematical Statement of Assignment Problem**.
Given the n X n matrix ($C_{ij}$) of real numbers, find a permutation *p* ( $p_i$, *i=1,... , n*) of the integers 1, 2, .. . *n* that minimizes

$$(\min)\sum_{i=1}^{n} c_{ip_i} \qquad (3)$$

*Example*

For the 4 X 4 matrix

$$C = \begin{bmatrix} 22 & 12 & 16 & 15 \\ 20 & 12 & 24 & 20 \\ 30 & 16 & 22 & 30 \\ 8 & 12 & 10 & 8 \end{bmatrix}$$

there are 24 possible permuatations (4! = 4 x 3 x 2 x 1 = 24).

The possible permuatations and the associated sums are given in table 1.

*Table 1*

| | P | $\sum_{i=1}^{n} a_{ip_i}$ |
|---|---|---|
| (1) | 1 2 3 4 | 64 |
| (2) | 1 2 4 3 | 74 |
| (3) | 1 3 2 4 | 70 |
| ... | | |
| (22) | 4 2 3 1 | 57 |
| ... | | |
| (24) | 4 3 2 1 | 63 |

Permutation 22 gives the smallest sum, that is, permutation 4 2 3 1 and the associated sum $\Sigma\ c_{14} + c_{22} + c_{33} + c_{41} =$ 15+12+22+8 which is 57.

## An Algorithm for the Assignment Problem

The so-called Hungarian Algorithm, named after Hungarian mathematician D.Konig who devised it, is used for solving model (2). This numerical method is based on minimizing the so-called opportunistic costs that are the result of non-engagement of the most efficient worker.

The method of assignment optimization is based on *n* usage of matrix C whose elements are quotens of the goal function, that is indicators of workers' efficiency in doing the jobs.

If *m* workers are needed for doing *n* jobs , efficiency matrix C is as follows:

$$C = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ c_{n1} & c_{n2} & \cdots & c_{nn} \end{bmatrix} \quad (3)$$

Whereby $c_{ij}$ stands for the costs *i* worker makes while doing *j* job.

Finding the optimal assignment within which the right worker is found for a specific job ensuring the minimal costs, is realized in three main phases:

#### Optimization Phase I

For each row of the matrix, find the smallest element and subtract it from every element in its row.

After that the similar approach is applied to the rows of the matrix, that is, for each row of the matrix the smallest element is found, which is then subtracted from every element in its row of the matrix.

The application of these steps in Phase I leads to forming matrix *C'* which has in each row at least one element $c_{ij}$ whose value is 0.

Matrix *C'* is the starting matrix from which we continue the optimization process in phases II and III.

#### Optimization Phase II

In matrix *C'* we identify the so-called starred zeros and primed zeros.

The simplest way of identifying starred and primed zeros is the following:

– The zeros within the rows are starred while the remaining zeros within the corresponding colums are primed.
– In the rows with a greater number of zeros, the starred zeros are assigned to the remaining rows starting from the rows with the smallest number of zeros paying attention to the fact that there can be only one starred zero in each row.

The optimal solution is found if the number of starred zeros is equal to the number of rows of the matrix, whereby the starred zeros identify the elements in the matrix whose sum represents the optimal assignment problem solving.

Otherwise we use vertical and horizontal lines to 'cover' the rows and columns in matrix *C'* which contain a zero value, whereby the minimal number of lines is used to cover the rows and columns with a zero value, and then we proceed to the third optimization phase.

#### Optimization Phase III

This phase of the process of finding the optimal solution is realized following the given steps:

– defining the minimal element in matrix *C'* which is not covered ,
– the minimal noncovered element in matrix *C'* is subtracted from all noncovered elements in the matrix,
– the minimal noncovered element in matrix *C'* is added to the twice covered elements in the matrix,
– the remaining, once covered elements in matrix *C'* remain unchanged.

After the third phase, matrix *C'* is modified and the same process is repeated in phases II and III until the number of primed zeros is equal to the number of rows in matrix *C'*.

## Example

We will show the assignment problem solving through an example given in the book Scientific Bases of Project Management (Naučne osnove upravljanja projektima):

There are four trucks (A, B, C, D) and four truck drivers for the transport of a larger amount of material (broken stone) needed for the construction of a part of the road. It has been measured and determined that each driver needs a specific period of time for each round, as is shown in table 2.

*Table 2*

| truck / driver | A | B | C | D |
|---|---|---|---|---|
| I | 22 | 12 | 16 | 15 |
| II | 20 | 12 | 24 | 20 |
| III | 30 | 16 | 22 | 30 |
| IV | 8 | 12 | 10 | 8 |

That is,

$$C = \begin{bmatrix} 22 & 12 & 16 & 15 \\ 20 & 12 & 24 & 20 \\ 30 & 16 & 22 & 30 \\ 8 & 12 & 10 & 8 \end{bmatrix}$$

After the first optimization phase, we have the following matric *C'*:

$$C' = \begin{bmatrix} 10 & 0^* & 2 & 3 \\ 8 & 0 & 10 & 8 \\ 14 & 0 & 4 & 14 \\ 0 & 4 & 0^* & 0 \end{bmatrix}$$

Since the number of starred zeros ($0^*$) is smaller than the number of the rows in matrix *C'* after the first optimization phase, the optimization process is continued through the second and then the third phase.

The optimal solution is found after the third iteration after which matrix *C' is as follows:*

$$C' = \begin{bmatrix} 7 & 2 & 0 & 0^* \\ 3 & 0^* & 6 & 3 \\ 9 & 0 & 0^* & 9 \\ 0^* & 9 & 1 & 0 \end{bmatrix}$$

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The optimal assignment of the drivers and the trucks and the needed working hours for the job is shown in table 3.

*Table 3*

| Driver | Truck | Time (min) |
|---|---|---|
| 1 | D | 15 |
| 2 | B | 12 |
| 3 | C | 22 |
| 4 | A | 8 |
| | Total time: | 57 |

No other assignment gives shorter time than this one (57 minutes).

## Conclusion

Optimization of the assignment of workers (and other resources) for doing particular jobs leads to a significant reduction of the costs, which is especially important when planning large projects.

The larger the number of alternatives (n), the larger the number of the possible assignment varieties (n! varieties). Nowadays computers can solve this problem of varieties and they can do taht rather fast. Yet, the Hungarian or Munkres' Assignment Algorithm is still an effective solution to this problem, even when we use a computer.

There are numerous varieties (modifications) of the above mentioned algorithm. However, we used our own version devised at the Faculty of Management in Zajecar for the example given in this thesis.

## References


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