APPLICATION OF LINEAR PROGRAMMING WITH FUZZY PARAMETERS IN MINING

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ABSTRACT. The fuzzy set theory has been applied in many fields, such as operations research, control theory, and management sciences, etc. In particular, an application of this theory in decision making problems is linear programming problems with fuzzy numbers. In this paper is given an example of usage of linear programming with fuzzy parameters for an process optimization in mining.

ПРИЛОЖЕНИЕ НА ЛИНЕЙНО ПРОГРАМИРАНЕ С НЕОПРЕДЕЛЕНИ (РАЗМИТИ) ПАРАМЕТРИ В МИННОТО ПРОИЗВОДСТВО

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РЕЗЮМЕ. Теорията на размитите (неопределени) множествата се прилага в много области на познанието, като например в научните изследвания за увеличаване на производствената ефективността, теорията за контрол и мениджмънта и т.н. По специално, прилагането на тази теория при решаване на възникнали проблеми е чрез линейното програмирани проблемите с неопределени (размити) числа. В доклада е представен пример с използване на линейно програмиране с неопределени (размити) параметри при оптимизиране ефективността на минното производството.

Introduction

Fuzzy linear programming is first formulated by Zimmerman. Recently, these problems are considered in several kinds, that is, it is possible that some coefficients of the problem in the objective function, technical coefficients, the right-hand size (RHS) coefficients or decision making variables be fuzzy number [3, 4, 5, 6, 7, 8, 9]. In this work, we focus on the linear programming problems with fuzzy numbers in objective function for optimization process in mining.

Fuzzy sets

Let X be a classical (*crisp*) set of objects, called the universe, whose generic elements are denoted by x. The membership in crisp subset of X is often viewed as characteristic function $\mu_A(x)$ from X to $\{0,1\}$ such that:

$$\mu_A(x) = \begin{cases} 1 & x \in A \\ 0 & x \notin A \end{cases} \quad (1)$$

where $\{0,1\}$ is called a valuation set.

If a valuation set is allowed to be the real interval [0,1], A is called a *fuzzy set* proposed by Zedeh. $\mu_A(x)$ is the degree of membership in of x in A . The closer the value of $\mu_A(x)$ is to 1, the more x belong to A. Therefore, A is completely characterized by the set of ordered pairs:

$$A = \{x, \mu_A(x) \mid x \in X\}$$

Fuzzy number

A fuzzy number A is convex normalized fuzzy set on the real line R such that:

1) It exist at least one $x_0 \in R$ with $\mu_A(x_0) = 1$.
2) $\mu_A(x)$ is piecewise continuous.

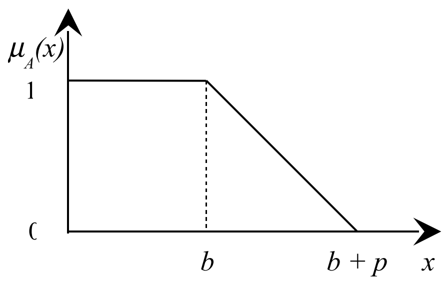

Fig. 1. Fuzzy number

$$\mu_A(x) = \begin{cases} 1 & x \le b \\ \dfrac{x-b}{p} & b < x \le b+p \\ 0 & b+p < x \end{cases} \quad (2)$$

Among the various types of fuzzy numbers, triangular and trapezoidal fuzzy numbers are the most important.

Fuzzy optimization

Fuzzy optimization problems can be stated and solved in many different ways. Usually the authors consider optimization problems of the form

$$\max/\min\ f(x);\ \text{subject to}\ x \in X,$$

where f or/and X are defined by fuzzy terms. Then they are search for crisp x^* which (in a certain) sense maximizes f under the (fuzzy) constraint X.

Linear Programming. A linear programming (LP) problem is defined as:

$$\begin{aligned} &\max z = cx \\ &\text{s.t. } Ax \le b \\ &x \ge 0 \end{aligned} \qquad (3)$$

where $c = (c_1, \dots, c_n)$, $b = (b_1, \dots, b_n)^T$, and $A = [a_{ij}]_{mxn}$.
Canonical form of linear programming problem is defined as:

$$\begin{aligned} &\max c^T x && \leftarrow \textit{objective function} \\ &\text{st. } Ax \le b && \leftarrow \textit{resource constraints} \\ &x \ge 0 && \leftarrow \textit{action variables} \end{aligned} \qquad (4)$$

$A \in R^{m\times n}$, $b \in R^m$, $c \in R^n$
Vector x^* is called a solution of LP problem if $cx^* \ge cx$ for all $x \in X$.

In the above problem, all of the parameters are crisp. Now, if some of the parameters be fuzzy numbers we obtain a fuzzy linear programming.

Fuzzy Linear Programming. In fuzzy linear programming (FLP) problems some or all coeficients can be fuzzy sets and the inequality relations between fuzzy sets can be given by certain fuzzy relation. In summary, the fuzzy optimization problem with fuzzy coefficients and fuzzy parameters can be mathematically expressed by:

$$\begin{aligned} &\max/\min\ z = \tilde{c}x \\ &\text{st. } \tilde{A}x\{\le, \ge, =\}\tilde{b} \end{aligned} \qquad (5)$$

Fuzzy linear programming problems with fuzzy numbers in objective function represent a particular fuzzy linear programming problems. There is a some special cases of fuzzy linear programming problems with fuzzy numbers in objective function:

a) Fuzzy tehnological coefficients

$$\begin{aligned} &\max\ c^T x \\ &\text{st. } \tilde{A}x \le b \end{aligned} \qquad (6)$$

b) Fuzzy coefficients on RHS

$$\begin{aligned} &\max\ c^T x \\ &\text{st. } Ax \le \tilde{b} \end{aligned} \qquad (7)$$

c) Fuzzy constraints

$$\begin{aligned} &\max\ c^T x \\ &\text{st. } \tilde{A}x \le \tilde{b} \end{aligned} \qquad (8)$$

Fuzzy solution

In the fuzzy environment fuzzy objective function and constraints is characterized by its membership functions. The decision in fuzzy environment can be viewed as the intersection of fuzzy constraints and fuzzy objective function.

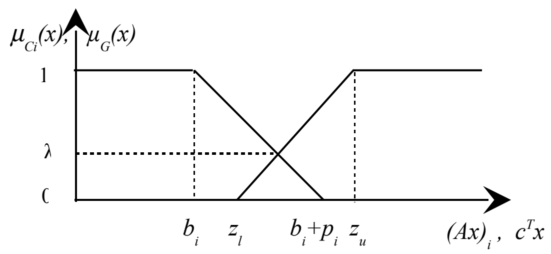

Fig. 2. Fuzzy solution

By using the definition of the fuzzy decision proposed by Bellman and Zedeh, we have:

$$\mu_D(x) = \min(\mu_G(x), \min_i(\mu_{Ci}(x))) \qquad (9)$$

Allowed fuzzy solution. We say that vector $x \in R^n$ is a feasible solution for (x1) if and only if x satisfies the constraints of the problem.

Optimal fuzzy solution. A feasible solution x^* is an optimal solution for (x1), if and only if x say that vector $x \in R^n$ is a feasible solution for (x1) and for all feasible solutions x we have $cx^* \ge cx$.
Consequently, the problem XX becomes to the following optimization problem:

$$\begin{aligned} &\max\ \lambda \\ &\mu_G(x) \ge \lambda \\ &\mu_{Ci}(x) \ge \lambda,\ 1 \le i \le m \\ &x \ge 0,\ \ 0 \le \lambda \le 1 \end{aligned} \qquad (10)$$

Fuzzy linear programming problems with fuzzy constraints

We consider a fuzzy linear programming problems with fuzzy constraints (c)

Assumption 1. $\tilde{a}_{ij}$ and $\tilde{b}_i$ are fuzzy number with the following membership function:

$$\mu_{a_{ij}}(x) = \begin{cases} 1 & x < a_{ij} \\ (a_{ij} + d_{ij} - x)/d_{ij} & a_{ij} \le x < a_{ij} + d_{ij} \\ 0 & x \ge a_{ij} + d_{ij} \end{cases}$$

and

$$\mu_{b_i}(x) = \begin{cases} 1 & x < b_i \\ (b_i + p_i - x)/p_i & b_i \le x < b_i + p_i \\ 0 & x \ge b_i + p_i \end{cases}$$

where $x \in$ R.

The fuzzy set of the ith constraint, C_i, is defined by

$$\mu_{Ci}(x) = \begin{cases} 0 & b_i < \sum_{j=1}^{n} a_{ij}x_j, \\ \left(bi - \sum_{j=1}^{n} a_{ij}x_j\right) / \left(\sum_{j=1}^{n} d_{ij}x_j + p_i\right) & \sum_{j=1}^{n} a_{ij}x_j \le b_i < \sum_{j=1}^{n} (a_{ij} + d_{ij})x_j + p_i, \\ 1 & b_i \ge \sum_{j=1}^{n} (a_{ij} + d_{ij})x_j, \end{cases}$$

For defuzification of problem (c), we first calculate the lower and upper bounds of the optimal values. The optimal values z_l and z_u can be defined by solving the following standard linear programming problems, for which we assume that all they have the finite optimal values.

The objective function takes values between z_l and z_u while tehnological coefficients take values between a_{ij} and $a_{ij}+d_{ij}$ and the right-hand side numbers take values between b_i and b_i+p_i.

The fuzzy set of optimal values, G, is defined as:

$$\mu_G(x) = \begin{cases} 0 & \sum_{j=1}^{n} c_jx_j < z_l, \\ \left(\sum_{j=1}^{n} c_jx_j - z_l\right) / (z_u - z_l) & z_l < \sum_{j=1}^{n} c_jx_j < z_u, \\ 1 & \sum_{j=1}^{n} c_jx_j > z_u, \end{cases}$$

By using the definition of the fuzzy decision proposed by Bellman and Zedeh, we have

$$\mu_D(x) = \min(\mu_G(x), \min_i(\mu_{c_i}(x)))$$

Consequently, the problem (x,x) becomes to the following optimization problem.

$$\max \lambda$$
$$\mu_G(x) \ge \lambda$$
$$\mu_{Ci}(x) \ge \lambda,\ 1 \le i \le m$$
$$x \ge 0,\quad 0 \le \lambda \le 1$$

By using (x.x) and (x.z), the problem (x.d) can be writen as:

$$\max \lambda$$
$$\sum_{j=1}^{n} c_jx_j \ge z_l + \lambda(z_u - z_l)$$
$$\sum_{j=1}^{n} (a_{ij}x_j + \lambda d_{ij}) \le b_i - \lambda p_i$$
$$x \ge 0,\quad 0 \le \lambda \le 1$$

An example of usage of fuzzy linear programming in mining

Quarry output is up to 15 m^3/h of rubble. Quarry produce two types of rubble:

- *A* (+ 8 – 16mm)
- *B* (+ 0 – 8mm)

Production costs should be up to 100€/h (6€/m^3 for rubble *A* and 7,5€/m^3 for rubble *B*).

It is expected benefit of 4,5€/m^3 from rubble *A* and 5€/m^3 from rubble *B*.

Determinate optimal output of rubble *A* and *B* in order to get peak of total benefit.

Solution. Objective function is:

$(\max)\ Z = 4.5X_1 + 5X_2$

Resource constraints:

$X_1 + X_2 \le 15$
$6X_1 + 7.5X_2 \le 100$

Modification. Modification is necessary in order to get the most real process of production. Dependenig of demand, output of rubble *A* can be increased up to 20%, and otput of rubble *B* up to 30%. Growth of the one rubble production can be realized by reduction of the other rubble or by growth of total output up to 10%. But, growth of total output up to 10% couse growth of total costs up to 13.7%.

We now have the new (modificated) resource constraints:

$$(1+0.2)x_1 + (1+0.3)x_2 \le 15 + 1.5$$
$$6x_1 + 7.5x_2 \le 100 + 13.7$$

By application of fuzzy technological coefficients and fuzzy coefficients on the right side (RHS) of the system, we get limited area ($C_1C_2C_3C_4$) which represents allowed fuzzy solution, instead of intersection point which represents the optimal solution.

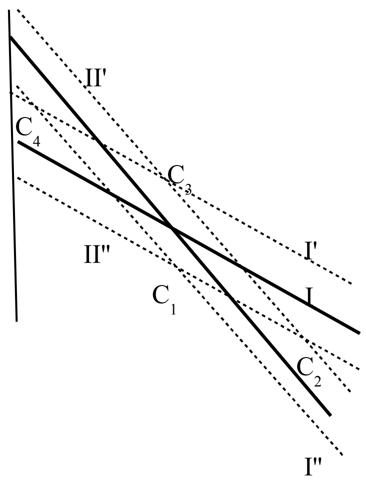

Fig. 3. Allowed fuzzy solution area

After solving the system (resource constraints) and determination of values of z_l and z_u (57.69 i 79.15), this system gets its final form:

$$\max \ \lambda$$
$$x_1 + x_2 \geq 57.69 + 21.46\lambda$$
$$(1 + 0.2\lambda)x_1 + (1 + 0.3\lambda)x_2 \leq 15 - 1.5\lambda$$
$$6x_1 + 7.5x_2 \leq 100 - 13.7\lambda$$
$$x_1, x_2 \geq 0, \quad 0 \leq \lambda \leq 1$$

Finally, it is determinated the value of the optimal fuzzy solution by iteration operation.

λ	z(λ)
0	57.6923
0.1	59.8381
0.2	61.9838
0.3	64.1296
0.3355	64.8914

Optimal fuzzy solution is obtained for λ^*=0.3355 and its value is $z(\lambda^*)$=64.8914.

Optimal fuzzy solution represents optimal total benefit (64.8914€/h) under above determined condition of production.

Recommended for publication by the Editorial staff of Section “Mining and Mineral Processing”

Conclusion

The fuzzy set theory has wide application, such as operations research, control theory, and management sciences, etc. The application of this theory in decision making problems is linear programming problems with fuzzy numbers. These problems are considered in several kinds, that is, it is possible that some coefficients of the problem in the objective function, tehnical coefficients, the right-hand size (RHS) coefficients or decision making variables be fuzzy number.

In this work, focus was on the linear programming problem with fuzzy numbers in objective function for optimization process in mining.

Based on quarry output of two types of rubble, cost of production and benefit it is obtained the objective function and resource constraints. Also, modification is done in order to get the most real process of production.

Solving this optimizational problem, it is determinated the value of the optimal fuzzy solution by iteration operation and optimal fuzzy solution is obtained for λ^*=0.3355. Its value is $z(\lambda^*)$=64.8914. This mean that optimal total benefit is 64.8914€/h under above determined condition of production.

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