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# НЕУТРАЛНИ УРАВНЕНИЯ С ПОЛИНОМНИ НЕЛИНЕЙНОСТИ ВЪЗНИКВАЩИ ПРИ ПРЕДАВАТЕЛНИТЕ ЛИНИИ

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РЕЗЮМЕ: Доказана е теорема за съществуване на решение на неутрални уравнения с полиномни нелинейности.

## ВЪВЕДЕНИЕ

Известно е, че в теорията на нелинейните вериги волт-амперните характеристики се апроксимират чрез различни нелинейни функции (вж. [1] - [3]) – полиноми, експоненти, хиперболични функции и техните комбинации. Много автори ([4] - [7]) при изследването на дълги линии без загуби разглеждат хиперболичната система

$$\frac{\partial u}{\partial x} = -L\frac{\partial i}{\partial t} \text{ и } \frac{\partial i}{\partial x} = -C\frac{\partial u}{\partial t}, \qquad (1)$$

където $i = i(t,x)$, $u = u(t,x)$ са съответно тока и напрежението в линията, а $L$ и $C$ са индуктивността и капацитета за единица дължина. Линията е с крайна дължина $l$, започваща при $x = 0$, което дава $u(0,t) = 0$ и е свързана с нелинеен резистивен елемент (например тунелен диод) при $x = l$, т.е. $i(l,t) = f(u(l,t) + E)$, където $E$ е напрежението на източника, а $I = f(V)$ е волт-амперната характеристика. В много случаи $f(V)$ е полином от трета степен от вида $f(V) = -aV + V^3$ (вж. [4] - [7]) или полином от по-висок ред [1], [2]. Ако в схемата е включен успоредно на нелинейния елемент кондензатор с капацитет $C_0$ горното гранично условие става

$$i(l,t) = f(u(l,t) + E) + C_0\frac{\partial u(l,t)}{\partial t}.$$

Тогава смесената начално-гранична задача за системата (1) може да се замени с чисто начална задача за едно функционално диференциално уравнение от неутрален тип ([4] - [7]), докато в първия случай то е само функционално уравнение. Общата теория на неутралните уравнения може да се намери в [8]-[11], но полиномни нелинейности се разглеждат само в частни случаи ([4] - [7]).

Основната цел на настоящата работа е да се формулират условия за съществуване и единственост на решения на функционално диференциални уравнения с полиномни нелинейности от общ вид в дясната страна. Нашите изследвания се основават върху подхода на неподвижните точки получен в [12]-[13].

Нека разгледаме една начална задача за неутралното функционално диференциално уравнение от първи ред:

$$\dot{u}(t) = F(u(\Delta_1(t)),\ldots,u(\Delta_m(t),\dot{u}(\gamma_1(t),\ldots,\dot{u}(\gamma_n(t)),t > 0,$$
$$u(t) = \varphi(t),\ \dot{u}(t) = \dot{\varphi}(t),\ t \le 0 \qquad (2)$$

където $F(u_1,\ldots,u_m,v_1,\ldots,v_n): R^{m+n} \to R^1$,

$\varphi(t),\dot{\varphi}(t): R^1_- \to R^1$, $R^1_+ = [0,\infty), R^1_- = (-\infty,0]$,
$\Delta_i(t): R^1_+ \to R^1$ $(i = 1,2,\ldots,m)$,

$\gamma_k(t): R^1_+ \to R^1$ $(k = 1,2,\ldots,n)$ са зададени функции.

Обикновено когато се търси глобално решение на (2) *F* удовлетворява условие от вида

$$|F(u_1,\ldots,u_m,v_1,\ldots,v_n)| \le a\left[\sum_{k=1}^{m}|u_k| + \sum_{s=1}^{n}|v_s|\right] \qquad (3)$$

Нашата цел е да се включат в разглежданията десни страни от общ тип:

$$F(u_1,\ldots,u_m,v_1,\ldots,v_n) = \sum_{s=1}^{k_1} a_s^{(1)}u_1^s + \sum_{s=1}^{k_2} a_s^{(2)}u_2^s + \ldots$$
$$+\sum_{s=1}^{k_m} a_s^{(m)}u_m^s + \sum_{k=1}^{n} b_k v_k \qquad (4)$$

където $a, a_s^{(l)}, b_k$ са дадени константи

.

Уравненията получени в [4] - [7] са частен случай на (4):

$$C_0[\dot{u}(t) + K\dot{u}(t-h)] + \left(\frac{1}{z} - g\right)u(t) - K\left(\frac{1}{z} + g\right)u(t-h) =$$
$$= -u^3(t) - Ku^3(t-h),$$

където $C_0, K, \frac{1}{z}, g$ и $h$ са дадени константи.

## ТЕОРЕМА ЗА СЪЩЕСТВУВАНЕ

Както обикновено полагаме $y(t) = \dot{u}(t)$ при $t > 0$ и $\psi(t) = \dot{\varphi}(t)$ при $t \le 0$. Тогава (2) става (предполагайки $y(0) = \varphi(0) = 0$):

$$y(t) = F\left(\int_0^{\Delta_1(t)} y(s)ds, ..., \int_0^{\Delta_m(t)} y(s)ds, y(\gamma_1(t)), ..., y(\gamma_n(t))\right), t > 0$$
$$y(t) = \psi(t), t \le 0. \quad (5)$$

Наистина $y(t - t_0) = u(t) - \varphi_0 - \varphi'_0(t - t_0)$ удовлетворява условията

$$y(0) = y(t_0 - t_0) = u(t_0) - \varphi_0 = 0,$$
$$y'(0) = u'(t_0) - \varphi'_0 = 0 \; (u(t_0) = \varphi_0, u'(t_0) = \varphi'_0).$$

**Теорема 1.** Нека се изпълнени следните условия:

1.1 функциите

$$\Delta_i(t), \gamma_k(t) : R_+^1 \to R^1 \; (i = 1,...,m; k = 1,...,n)$$

са непрекъснати и $\Delta_i(0) \le 0$, $\gamma_k(0) \le 0$ и

$$t - \Delta_i(t) \ge \Delta_0 > 0, \; t - \gamma_k(t) \ge \gamma_0 > 0,$$

където $\Delta_0, \gamma_0 > 0$ са константи;

1.2 функцията

1.3

$$F(u_1, ..., u_m, v_1, ..., v_n) : R^{m+n} \to R^1$$

е непрекъсната и удовлетворява условията:

1.2.1 $\psi(t) = \dot{\varphi}(t)$ е непрекъсната и у

довлетворява условието за съгласуваност

$$\psi(0) = F\left(\int_0^{\Delta_1(0)} \psi(s)ds, ..., \int_0^{\Delta_m(0)} \psi(s)ds, \psi(\gamma_1(0)), ..., \psi(\gamma_n(0))\right);$$

1.2.2 $$|F(u_1, ..., u_m, v_1, ..., v_n)| \le$$
$$\le a_1\left(\sum_{s=0}^{k_1} |u_1|^s + ... + \sum_{s=0}^{k_m} |u_m|^s\right) + a_2 \sum_{s=1}^{n} |v_s|,$$

където $a_1$ и $a_2$ са положителни константи;

1.2.3 $$|F(u_1, ..., u_m, v_1, ..., v_n)| - |F(\tilde{u}_1, ..., \tilde{u}_m, \tilde{v}_1, ..., \tilde{v}_n)| \le$$
$$\le b_1\left(\sum_{s=1}^{k_1} |u_1^s - \tilde{u}_1^s| + ... + \sum_{s=1}^{k_m} |u_m^s - \tilde{u}_m^s|\right) + b_2 \sum_{s=1}^{n} |v_s - \tilde{v}_s|$$

Тогава (5) има единствено непрекъснато решение.

**Доказателство:** Разглеждаме множеството $X$ от всички непрекъснати функции $f(t) : [0, T_0] \to R^1$, които съвпадат с $\psi(t)$ при $t \le 0$.

Въвеждаме фамилия от псевдометрики

$$A = \{\rho_\lambda(f, \tilde{f}) : \lambda \in [0, \infty)\},$$

където $\rho_\lambda(f, \tilde{f}) = \sup\{e^{-\lambda t} | f(t) - \tilde{f}(t)| : t \in [0, T_0]\}$.

Нека напомним, че при $t \le 0$ $f(t) = \psi(t)$ и $\tilde{f}(t) = \psi(t)$.

Тогава $X$ снабдено с фамилията $A$ става едно равномерно пространство $(X, A)$.

Въвеждаме множеството

$$M = \{f \in (X, A) : |f(t)| \le Ae^{\lambda t}, t \in [0, T_0]\},$$

като $A$ е фиксирана константа, която не зависи от $f$.

Лесно се проверява, че $(Tf)(t)$ е непрекъсната функция върху $R^1$.

Първо ще покажем, че операторът $T$, дефиниран чрез десната страна на (5) е свиващ:

$$(Tf)(t) = \begin{cases} F\left(\int_0^{\Delta_1(t)} f(s)ds, ..., \int_0^{\Delta_m(t)} f(s)ds, f(\gamma_1(t)), ..., f(\gamma_n(t))\right), t \in [0, T_0] \\ \psi(t), t \le 0. \end{cases}$$

Наистина за всеки $f, \tilde{f} \in M$ имаме при $t \in [0, T_0]$, за които $\Delta_k(t) > 0$:

$$|(Tf)(t) - (T\tilde{f})(t)| \le b_1\left[\sum_{s=1}^{k_1}\left|\left(\int_0^{\Delta_1(t)} f(\tau)d\tau\right)^s - \left(\int_0^{\Delta_1(t)} \tilde{f}(\tau)d\tau\right)^s\right| + ... +\right.$$

$$\left.+\sum_{s=1}^{k_m}\left|\left(\int_0^{\Delta_m(t)} f(\tau)d\tau\right)^s - \left(\int_0^{\Delta_m(t)} \tilde{f}(\tau)d\tau\right)^s\right|\right] + b_2\sum_{s=1}^{n} |f(\gamma_s(t)) - \tilde{f}(\gamma_s(t))| \le$$

$$\le b_1\left[\sum_{s=1}^{k_1}\left|\int_0^{\Delta_1(t)} f(\tau)d\tau - \int_0^{\Delta_1(t)} \tilde{f}(\tau)d\tau\right| s \left|\int_0^{\Delta_1(t)} \bar{f}(\tau)d\tau\right|^{s-1} + ... +\right.$$

$$+\sum_{s=1}^{k_m}\left|\int_0^{\Delta_m(t)} f(\tau)d\tau - \int_0^{\Delta_m(t)}\tilde{f}(\tau)d\tau\right| s \left|\int_0^{\Delta_m(t)}\bar{f}(\tau)d\tau\right|^{s-1} +$$

$$+ b_2\sum_{s=1}^{n} e^{-\lambda\gamma_s(t)} \,|\, f(\gamma_s(t)) - \tilde{f}(\gamma_s(t))\,|\, e^{\lambda\gamma_s(t)} \le$$

$$\le b_1\left[\sum_{s=1}^{k_1} sA^{s-1}\left|\int_0^{\Delta_1(t)} e^{\lambda\tau}d\tau\right|^{s-1}\left|\int_0^{\Delta_1(t)} |\, f(\tau) - \tilde{f}(\tau)\,|\, e^{-\lambda\tau}.e^{\lambda\tau}d\tau\right| + \ldots\right.$$

$$\left.+\sum_{s=1}^{k_m} sA^{s-1}\left|\int_0^{\Delta_m(t)} e^{\lambda\tau}d\tau\right|^{s-1}\left|\int_0^{\Delta_m(t)} |\, f(\tau) - \tilde{f}(\tau)\,|\, e^{-\lambda\tau}.e^{\lambda\tau}d\tau\right|\right] +$$

$$+ b_2\rho_\lambda(f,\tilde{f})\sum_{s=1}^{n} e^{-\lambda\gamma_s(t)} \le$$

$$\le b_1\rho_\lambda(f,\tilde{f})\left[\sum_{s=1}^{k_1} sA^{s-1}\left|\int_0^{\Delta_1(t)} e^{\lambda\tau}d\tau\right|^{s} + \ldots + \sum_{s=1}^{k_m} sA^{s-1}\left|\int_0^{\Delta_m(t)} e^{\lambda\tau}d\tau\right|^{s}\right] +$$

$$+ b_2\rho_\lambda(f,\tilde{f})\sum_{s=1}^{n} e^{-\lambda\gamma_s(t)} \le \quad (k = \max\{k_1, k_2, \ldots, k_m\})$$

$$\le b_1\rho_\lambda(f,\tilde{f})\left[\sum_{s=1}^{k_1} sA^{s-1}\left|\frac{e^{\lambda\Delta_1(t)} - 1}{\lambda}\right|^{s} + \ldots + \sum_{s=1}^{k_m} sA^{s-1}\left|\frac{e^{\lambda\Delta_m(t)} - 1}{\lambda}\right|^{s}\right] +$$

$$+ b_2\rho_\lambda(f,\tilde{f})\sum_{s=1}^{n} e^{-\lambda t + \lambda\gamma_s(t)}.e^{\lambda t} \le$$

$$\le \rho_\lambda(f,\tilde{f})\left[b_1\left(\sum_{s=1}^{k_1} sA^{s-1}e^{\lambda st}\left(\frac{e^{-\lambda t + \lambda\Delta_1(t)}}{\lambda}\right)^{s} + \ldots\right.\right.$$

$$\left.\left.+\sum_{s=1}^{k_m} sA^{s-1}e^{\lambda st}\left(\frac{e^{-\lambda t + \lambda\Delta_m(t)}}{\lambda}\right)^{s}\right) + e^{\lambda t}b_2\sum_{1}^{n} e^{-\lambda\gamma_0}\right] \le \quad (-\lambda t + \lambda\Delta_i(t) \le -\lambda\Delta_0)$$

$$\le \rho_\lambda(f,\tilde{f})\left[b_1 m\sum_{s=1}^{k} sA^{s-1}\frac{e^{-\lambda\Delta_0 s}}{\lambda^s}e^{\lambda st} + ne^{\lambda t}b_2 e^{-\lambda\gamma_0}\right].$$

Умножаваме горните неравенства с $e^{-\lambda t}$ и тогава $e^{-\lambda t}\,|\,(Tf)(t) - (T\tilde{f}(t)\,| \le$

$$\le \rho_\lambda(f,\tilde{f})\left[b_1 m\sum_{s=1}^{k} sA^{s-1}\frac{e^{-\lambda\Delta_0 s}}{\lambda^s}e^{\lambda(s-1)T_0} + nb_2 e^{-\lambda\gamma_0}\right] \le$$

$$\le \rho_\lambda(f,\tilde{f})\left[b_1 m\frac{e^{-\lambda\Delta_0}}{\lambda}\sum_{s=1}^{k} s\left[\frac{Ae^{-\lambda\Delta_0 + \lambda T_0}}{\lambda}\right]^{s-1} + nb_2 e^{-\lambda\gamma_0}\right] \equiv$$

$$\equiv \rho_\lambda(f,\tilde{f})B(\lambda) \quad\Rightarrow\quad \rho_\lambda(Tf,T\tilde{f}) \le B(\lambda)\rho_\lambda(f,\tilde{f}).$$

За достатъчно голямо $\lambda$, $B(\lambda) < 1$ стига да е изпълнено неравенството $T_0 - \Delta_0 \le 0$. От това следва, че $T$ е свиващ.

Ще покажем, че от $f \in M \;\Rightarrow\; Tf \in M$. Наистина ($\Delta_k > 0$):

$$|\,(Tf)(t)\,| \le a_1\left(\sum_{s=1}^{k_1}\left|\int_0^{\Delta_1(t)} f(\tau)d\tau\right|^{s} + \ldots + \sum_{s=1}^{k_m}\left|\int_0^{\Delta_m(t)} f(\tau)d\tau\right|^{s}\right) +$$

$$+ a_2\sum_{s=1}^{n} |\, f(\gamma_s(t))\,| \le$$

$$\le a_1\left(\sum_{s=1}^{k_1} A^s\left(\frac{e^{\lambda\Delta_1(t)} - 1}{\lambda}\right)^{s} e^{\lambda st} + \ldots + \sum_{s=1}^{k_m} A^s\left(\frac{e^{\lambda\Delta_m(t)} - 1}{\lambda}\right)^{s} e^{\lambda st}\right) +$$

$$+ a_2 e^{\lambda t}\sum_{s=1}^{n} e^{-\lambda t + \lambda\gamma_s(t)}A \le$$

$$\le a_1 m\sum_{s=1}^{k}\left(\frac{A}{\lambda}\right)^{s}\left(e^{-\lambda\Delta_0 + \lambda T_0}\right)^{s} + a_2 e^{\lambda t}Ane^{-\lambda\gamma_0}.$$

Умножаваме последните неравенства с $e^{-\lambda t}$:

$$e^{-\lambda t}\,|\,(Tf)(t)\,| \le$$

$$\le a_1 m\sum_{s=1}^{k}\left(\frac{A}{\lambda}\right)^{s}\left(e^{-\lambda\Delta_0 + \lambda T_0}\right)^{s} e^{-\lambda t} + na_2 Ae^{-\lambda\gamma_0} \le$$

$$\le a_1 m\sum_{s=1}^{k}\left(\frac{A}{\lambda}\right)^{s}\left(e^{-\lambda\Delta_0 + \lambda T_0}\right)^{s} + na_2 Ae^{-\lambda\gamma_0} \le A,$$

което винаги е удовлетворено за достатъчно големи $\lambda$ и $T_0 - \Delta_0 \le 0$.

За да са изпълнени всички условия от теоремата от [12] трябва да имаме такъв елемент, че

$$\rho_{j^k(\lambda)}(x_0, Tx_0) \le Q < \infty \quad (k = 1,2,\ldots)$$

Тук изображението $j : A \to A$ (от [12]) е $j(\lambda) = \lambda$. В качеството на $x_0$ можем да изберем

$$\overline{0}(t) = \begin{cases} \psi(0), & t \geq 0 \\ \psi(t), & t \leq 0 \end{cases}.$$

Тогава получаваме

$$\rho_{j^k(\lambda)}(x_0, Tx_0) =$$

$$= \sup\{|\psi(0) - F(\Delta_1(t)\psi(0),\ldots,\Delta_m(t)\psi(0),\psi(0),\ldots,\psi(0))|e^{-\lambda t} : t \in [0,T_0]\} < \infty$$

Последният супремум съществува понеже $|\Delta_i(t)| \leq |t - \Delta_0|$ и следователно

$$e^{-\lambda t}\sum_{s=1}^{k_i}(|\Delta_i(t)|\psi(0))^s < \infty, \ (i = 1,2,\ldots,m).$$

Следователно $T$ има единствена неподвижна точка, която е решение (5).

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# NEUTRAL EQUATIONS WITH POLYNOMIAL NONLINEARITIES ARISING IN TRANSMISSION LINES


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ABSTRACT: An existence theorem for neutral equations with polynominal nonlinearities is proved.


## INTRODUCTION

It is known that in the theory of nonlinear circuits $V-I$ characteristic curve can be approximated by various nonlinear functions (cf. [1]-[3]) - polynoms, exponential functions, hyperbolic functions and their combinations. In [4]-[7] the authors in their endevour to investigate the lossless transmission lines considers the hyperbolic system

$$\frac{\partial u}{\partial x}=-L\frac{\partial i}{\partial t} \text{ и } \frac{\partial i}{\partial x}=-C\frac{\partial u}{\partial t}, \qquad (1)$$

where $i=i(t,x)$, $u=u(t,x)$ are the current and the voltage respectively, while $L$ is the series inductance, and $C$ is the parallel capacitance per unit length of the line. The line is shorted at $x=0$ which implies $u(0,t)=0$ and it is connected with a nonlinear element (for instance tunnel diode) at $x=l$ ($l$ is the length of the line), that is, $i(l,t)=f(u(l,t)+E)$, where $E$ is the bias voltage, and $I=f(V)$ is the $V-I$ characteristic curve. In many cases $f(V)$ is third degree polynom of the type $f(V)=-aV+V^3$ (cf. [4]-[7]) or polynom of higher order [1], [2]. If the parallel capacitance $C_0$ is considered in the circuit the above boundary value condition becomes

$$i(l,t)=f(u(l,t)+E)+C_0\frac{\partial u(l,t)}{\partial t}.$$

Then the mixed initial-boundary problem for system (1) can be replaced by a pure initial value problem for a functional differential equation of neutral type ([4]-[7]) while in the first one obtains only functional equations. The general theory of neutral equations can be found in [8]-[11] but the polynomial nonlinearities only in particular cases are considered [4]-[7].

The main purpose of the present paper is to formulate conditions for the existence and uniqueness of solutions of neutral functional differential equations with polynomial nonlinearities in the right-hand sides in general case. Our investigations are based on fixed point approach obtained in [12]-[13].

Let us consider an initial value problem for the neutral functional differential equation of first order

$$\dot u(t)=F(u(\Delta_1(t)),,...,u(\Delta_m(t),\dot u(\gamma_1(t),...,\dot u(\gamma_n(t)),t>0,\ u(t)=\varphi(t),\dot u(t)=\dot\varphi(t),t\le 0 \qquad (2)$$

where $F(u_1,...,u_m,v_1,...,v_n):R^{m+n}\to R^1$,

$\varphi(t),\dot\varphi(t):R^1_-\to R^1$, $R^1_+=[0,\infty),R^1_-=(-\infty,0]$,

$\Delta_i(t):R^1_+\to R^1$ $(i=1,2,...,m)$,

$\gamma_k(t):R^1_+\to R^1$ $(k=1,2,...,n)$ are prescribed functions.

Usually when one look for global solution of (2) $F$ has to satisfy the condition of the type

$$| F(u_1,...,u_m,v_1,...,v_n) | \le a\left[\sum_{k=1}^{m} | u_k | + \sum_{s=1}^{n} | v_s |\right] \quad (3)$$

Our goal is to include in the consicleration the right-hand sides of general type:

$$F(u_1,...,u_m,v_1,...,v_n) = \sum_{s=1}^{k_1} a_s^{(1)} u_1^s + \sum_{s=1}^{k_2} a_s^{(2)} u_2^s + ...$$

$$+ \sum_{s=1}^{k_m} a_s^{(m)} u_m^s + \sum_{k=1}^{n} b_k v_k \quad (4)$$

where $a, a_s^{(l)}, b_k$ are prescribed constants.

The equations obtained in [4] - [7] are particular cases of (4):

$$C_0[\dot{u}(t) + K\dot{u}(t-h)] + \left(\frac{1}{z} - g\right)u(t) - K\left(\frac{1}{z} + g\right)u(t-h) = -u^3(t) - Ku^3(t-h),$$

where $C_0, K, \frac{1}{z}, g$ and $h$ are prescribed constants.

EXISTENCE THEOREM

As usually we put $y(t) = \dot{u}(t)$ for $t > 0$ and $\psi(t) = \dot{\varphi}(t)$ for $t \le 0$. Then (2) becomes assuming $y(0) = \varphi(0) = 0$):

$$y(t) = F\left(\int_0^{\Delta_1(t)} y(s)ds,..., \int_0^{\Delta_m(t)} y(s)ds, y(\gamma_1(t)),..., y(\gamma_n(t))\right), t > 0 \;\; y(t) = \psi(t), t \le 0. \quad (5)$$

Indeed $y(t - t_0) = u(t) - \varphi_0 - \varphi'_0(t - t_0)$ satisfies

Condition
s

$y(0) = y(t_0 - t_0) = u(t_0) - \varphi_0 = 0$ and

$y'(0) = u'(t_0) - \varphi'_0 = 0 \; (u(t_0) = \varphi_0, u'(t_0) = \varphi'_0)$.

**Theorem 1.** Let the following conditions be fulfilled:

1.4 Functions

1.5

$$\Delta_i(t), \gamma_k(t) : R_+^1 \to R^1 \;\; (i = 1,...,m; k = 1,...,n)$$

are continuous and $\Delta_i(0) \le 0$, $\gamma_k(0) \le 0$, and

$$t - \Delta_i(t) \ge \Delta_0 > 0, \; t - \gamma_k(t) \ge \gamma_0 > 0,$$

where $\Delta_0$ and $\gamma_0 > 0$ are constants;

1.6 the function

$$F(u_1,...,u_m,v_1,...,v_n) : R^{m+n} \to R^1$$

is continuous and satisfies the conditions:

1.6.1 $\psi(t) = \dot{\varphi}(t)$ is continuous and satisfies conformity condition

1.6.2

$$\psi(0) = F\left(\int_0^{\Delta_1(0)} \psi(s)ds,..., \int_0^{\Delta_m(0)} \psi(s)ds, \psi(\gamma_1(0)),..., \psi(\gamma_n(0))\right);$$

1. $|F(u_1,...,u_m,v_1,...,v_n)| \le$
2.

$$\le a_1\left(\sum_{s=0}^{k_1}|u_1|^s + ... + \sum_{s=0}^{k_m}|u_m|^s\right) + a_2\sum_{s=1}^{n}|v_s|,$$

where $a_1$ and $a_2$ are positive constants;
1.2.3

$$|F(u_1,...,u_m,v_1,...,v_n)| - |F(\widetilde{u}_1,...,\widetilde{u}_m,\widetilde{v}_1,...,\widetilde{v}_n)| \le \le b_1\left(\sum_{s=1}^{k_1}|u_1^s - \widetilde{u}_1^s| + ... + \sum_{s=1}^{k_m}|u_m^s - \widetilde{u}_m^s|\right) + b_2\sum_{s=1}^{n}|v_s - \widetilde{v}_s|$$

Then (5) has a unique continuous solution.

**Proof.** Consider the set $X$ of all continuous functions $f(t):[0,T_0]\to R^1$ which coincide with $\psi(t)$ for $t \le 0$.

Introduce a family of pseudometrics

$$A = \{\rho_\lambda(f,\widetilde{f}): \lambda \in [0,\infty)\},$$ where

$$\rho_\lambda(f,\widetilde{f}) = \sup\{e^{-\lambda t}|f(t) - \widetilde{f}(t)|: t \in [0,T_0]\}.$$

Recall that for $t \le 0$ $f(t) = \psi(t)$ and $\widetilde{f}(t) = \psi(t)$.

Then the set $X$ endowed with the family $A$ becomes a uniform space $(X, A)$.

Introduce the set

$$M = \{f \in (X, A): |f(t)| \le Ae^{\lambda t}, t \in [0,T_0]\},$$

where $A$ is a fixed constant which does not depend on $f$.

It is easy to verify that $(Tf)(t)$ is a continuous function on $R^1$.

First we show that the operator $T$ defined by the right-hand side of (5) is contractive:

$$(Tf)(t) = \begin{cases} F\left(\int_0^{\Delta_1(t)} f(s)ds,..., \int_0^{\Delta_m(t)} f(s)ds, f(\gamma_1(t)),...,f(\gamma_n(t))\right), t \in [0,T_0] \\ \psi(t), t \le 0. \end{cases}$$

Indeed for every $f, \widetilde{f} \in M$ we have $t \in [0,T_0]$ for which $\Delta_k(t) > 0$:

$$|(Tf)(t) - (T\widetilde{f}(t)| \le b_1\left[\sum_{s=1}^{k_1}\left|\left(\int_0^{\Delta_1(t)} f(\tau)d\tau\right)^s - \left(\int_0^{\Delta_1(t)} \widetilde{f}(\tau)d\tau\right)^s\right| + ... +\right.$$

$$\left.+ \sum_{s=1}^{k_m}\left|\left(\int_0^{\Delta_m(t)} f(\tau)d\tau\right)^s - \left(\int_0^{\Delta_m(t)} \widetilde{f}(\tau)d\tau\right)^s\right|\right] + b_2\sum_{s=1}^{n}|f(\gamma_s(t)) - \widetilde{f}(\gamma_s(t))| \le$$

$$\leq b_1\left[\sum_{s=1}^{k_1}\left|\int_0^{\Delta_1(t)} f(\tau)d\tau - \int_0^{\Delta_1(t)}\tilde{f}(\tau)d\tau\right| s \left|\int_0^{\Delta_1(t)}\bar{f}(\tau)d\tau\right|^{s-1} + \ldots +\right.$$

$$+\sum_{s=1}^{k_m}\left|\int_0^{\Delta_m(t)} f(\tau)d\tau - \int_0^{\Delta_m(t)}\tilde{f}(\tau)d\tau\right| s \left|\int_0^{\Delta_m(t)}\bar{f}(\tau)d\tau\right|^{s-1} +$$

$$+ b_2\sum_{s=1}^{n} e^{-\lambda\gamma_s(t)} \,|\, f(\gamma_s(t)) - \tilde{f}(\gamma_s(t))\,|\, e^{\lambda\gamma_s(t)} \leq$$

$$\leq b_1\left[\sum_{s=1}^{k_1} sA^{s-1}\left|\int_0^{\Delta_1(t)} e^{\lambda\tau}d\tau\right|^{s-1}\left|\int_0^{\Delta_1(t)} |\, f(\tau) - \tilde{f}(\tau)\,|\, e^{-\lambda\tau}.e^{\lambda\tau}d\tau\right| + \ldots\right.$$

$$\left.+\sum_{s=1}^{k_m} sA^{s-1}\left|\int_0^{\Delta_m(t)} e^{\lambda\tau}d\tau\right|^{s-1}\left|\int_0^{\Delta_m(t)} |\, f(\tau) - \tilde{f}(\tau)\,|\, e^{-\lambda\tau}.e^{\lambda\tau}d\tau\right|\right] +$$

$$+ b_2\rho_\lambda(f,\tilde{f})\sum_{s=1}^{n} e^{-\lambda\gamma_s(t)} \leq$$

$$\leq b_1\rho_\lambda(f,\tilde{f})\left[\sum_{s=1}^{k_1} sA^{s-1}\left|\int_0^{\Delta_1(t)} e^{\lambda\tau}d\tau\right|^{s} + \ldots + \sum_{s=1}^{k_m} sA^{s-1}\left|\int_0^{\Delta_m(t)} e^{\lambda\tau}d\tau\right|^{s}\right] + \; + b_2\rho_\lambda(f,\tilde{f})\sum_{s=1}^{n} e^{-\lambda\gamma_s(t)} \leq \qquad (k = \max\{k_1,k_2,\ldots,k_m\})$$

$$\leq b_1\rho_\lambda(f,\tilde{f})\left[\sum_{s=1}^{k_1} sA^{s-1}\left|\frac{e^{\lambda\Delta_1(t)}-1}{\lambda}\right|^{s} + \ldots + \sum_{s=1}^{k_m} sA^{s-1}\left|\frac{e^{\lambda\Delta_m(t)}-1}{\lambda}\right|^{s}\right] +$$

$$+ b_2\rho_\lambda(f,\tilde{f})\sum_{s=1}^{n} e^{-\lambda t + \lambda\gamma_s(t)}.e^{\lambda t} \leq$$

$$\leq \rho_\lambda(f,\tilde{f})\left[b_1\left(\sum_{s=1}^{k_1} sA^{s-1}e^{\lambda st}\left(\frac{e^{-\lambda t+\lambda\Delta_1(t)}}{\lambda}\right)^{s} + \ldots + \sum_{s=1}^{k_m} sA^{s-1}e^{\lambda st}\left(\frac{e^{-\lambda t+\lambda\Delta_m(t)}}{\lambda}\right)^{s}\right) + e^{\lambda t}b_2\sum_{1}^{n} e^{-\lambda\gamma_0}\right] \leq \; (-\lambda t + \lambda\Delta_i(t) \leq -\lambda\Delta_0)$$

$$\leq \rho_\lambda(f,\tilde{f})\left[b_1 m\sum_{s=1}^{k} sA^{s-1}\frac{e^{-\lambda\Delta_0 s}}{\lambda^s}e^{\lambda st} + ne^{\lambda t}b_2 e^{-\lambda\gamma_0}\right].$$

We multiply the previous inequality with $e^{-\lambda t}$ and then $e^{-\lambda t}\,|\,(Tf)(t) - (T\tilde{f}(t)\,| \leq$

$$\leq \rho_\lambda(f,\tilde{f})\left[b_1 m\sum_{s=1}^{k} sA^{s-1}\frac{e^{-\lambda\Delta_0 s}}{\lambda^s}e^{\lambda(s-1)T_0} + nb_2 e^{-\lambda\gamma_0}\right] \leq$$

$$\leq \rho_\lambda(f,\tilde{f})\left[b_1 m\frac{e^{-\lambda\Delta_0}}{\lambda}\sum_{s=1}^{k} s\left[\frac{Ae^{-\lambda\Delta_0+\lambda T_0}}{\lambda}\right]^{s-1} + nb_2 e^{-\lambda\gamma_0}\right] \equiv$$

$$\equiv \rho_\lambda(f,\tilde{f})B(\lambda) \quad \Rightarrow \quad \rho_\lambda(Tf,T\tilde{f}) \leq B(\lambda)\rho_\lambda(f,\tilde{f}).$$

For sufficiently large $\lambda$ one can see that $B(\lambda) < 1$ provided $T_0 - \Delta_0 \leq 0$. This implies that $T$ is contractive.

We show $f \in M \Rightarrow Tf \in M$. Indeed ($\Delta_k > 0$):

$$|(Tf)(t)| \le a_1 \left( \sum_{s=1}^{k_1} \left| \int_0^{\Delta_1(t)} f(\tau)d\tau \right|^s + \ldots + \sum_{s=1}^{k_m} \left| \int_0^{\Delta_m(t)} f(\tau)d\tau \right|^s \right) +$$

$$+ a_2 \sum_{s=1}^{n} |f(\gamma_s(t))| \le$$

$$\le a_1 \left( \sum_{s=1}^{k_1} A^s \left( \frac{e^{\lambda \Delta_1(t)} - 1}{\lambda} \right)^s e^{\lambda st} + \ldots + \sum_{s=1}^{k_m} A^s \left( \frac{e^{\lambda \Delta_m(t)} - 1}{\lambda} \right)^s e^{\lambda st} \right) +$$

$$+ a_2 e^{\lambda t} \sum_{s=1}^{n} e^{-\lambda t + \lambda \gamma_s(t)} A \le$$

$$\le a_1 m \sum_{s=1}^{k} \left( \frac{A}{\lambda} \right)^s \left( e^{-\lambda \Delta_0 + \lambda T_0} \right)^s + a_2 e^{\lambda t} A n e^{-\lambda \gamma_0} .$$

Multiply the last inequality $e^{-\lambda t}$:

$$e^{-\lambda t} |(Tf)(t)| \le$$

$$\le a_1 m \sum_{s=1}^{k} \left( \frac{A}{\lambda} \right)^s \left( e^{-\lambda \Delta_0 + \lambda T_0} \right)^s e^{-\lambda t} + n a_2 A e^{-\lambda \gamma_0} \le$$

$$\le a_1 m \sum_{s=1}^{k} \left( \frac{A}{\lambda} \right)^s \left( e^{-\lambda \Delta_0 + \lambda T_0} \right)^s + n a_2 A e^{-\lambda \gamma_0} \le A,$$

which is always satisfied for sufficiently large $\lambda$ and $T_0 - \Delta_0 \le 0$.

In order to be satified all conditions of the fixed point theorem [12] we have to find such an element $x_0$ that

$$\rho_{j^k(\lambda)}(x_0, Tx_0) \le Q < \infty \quad (k = 1,2,\ldots)$$

Here the map $j : A \to A$ (cf. [12]) is $j(\lambda) = \lambda$. One can choose $x_0$

$$x_0(t) = \begin{cases} \psi(0), & t \ge 0 \\ \psi(t), & t \le 0 \end{cases}.$$

Then we obtain

$$\rho_{j^k(\lambda)}(x_0, Tx_0) =$$

$$= \sup\{ |\psi(0) - F(\Delta_1(t)\psi(0),\ldots,\Delta_m(t)\psi(0),\psi(0),\ldots,\psi(0))| e^{-\lambda t} : t \in [0,T_0] \} < \infty$$

The last supremum exists because

$|\Delta_i(t)| \le |t - \Delta_0|$ and consequently

$$e^{-\lambda t} \sum_{s=1}^{k_i} (|\Delta_i(t)| \psi(0))^s < \infty, \quad (i = 1,2,\ldots,m).$$

Therefore $T$ has a unique fixed point [12], which is a solution of (5).

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