

## DYNAMIC BEHAVIOR OF SERVO CONTROLLED HYDROSTATIC POWER TRANSMISSION SYSTEM WITH LONG TRANSMISSION LINE

Novak Nedić, Radovan Petrović, Ljubisa Dubonjić

University of Kragujevac, Faculty of Mechanical Engineering Kraljevo, 36 000Kraljevo, Serbia

e-mail: nedic.n@maskv.edu.yu , petrovic.r@maskv.edu.yu , dubonjic.lj@maskv.edu.yu

### ABSTRACT

Hydrostatic power transmission systems with long transmission line between pump and motor have more complex dynamic behavior than normal. Moreover, including closed loop in such systems it demands that design must be very careful in selection of control parameters and transmission line. The paper presents modeling and simulation analysis of dynamic behavior of servo hydrostatic power transmission system with long transmission line in frequency and time domen.

Keywords: servo hydrostatic power transmission systems, dynamic behavior, long transmission line

### INTRODUCTION

The pump is connected to the motor via line (pipe, tube) that could in some applications, such as mining, construction machines, heavy machines and remote control systems, be very long. In accordance with their strong behavior demands, for example, a motor is commanded to change to a different speed (from very low speed to very high speed and vice-versa) for short time. In such case the dynamics of the system must to be considered, that is, the dynamics of the all elements coupled in the system (pump, line, motor etc.) must to be considered. Moreover, having servo control of that system, the dynamics of the coupled system becomes more important. Authors of this paper assumed usual type of a servo hydrostatic power transmission system with a variable-displacement pump and a fixed- displacement motor as shown in Fig.1. The coupled subsystem pump-line-motor is shown in Fig.2.

The modeling of fluid transmission line has received a great deal of attention over the past few decades (from about 1950) [1-3] and there are several hundreds publication that could be quoted relating to different theories and applications for air, water and oil hydraulic systems [4-6]. The effect of transmission line dynamics on the dynamic behavior of fluid power systems was studied by J. Watton [7]. Simulation and experimental analysis of dynamics and control of pump controlled motor are presented in [8-10]. Some results in regard to dynamics of pump controlled motor with long transmission line were presented in [11-13]. This paper presents analysis on dynamic behavior of servo pump controlled motor with long transmission line based on different mathematical line models in frequency and time domen.

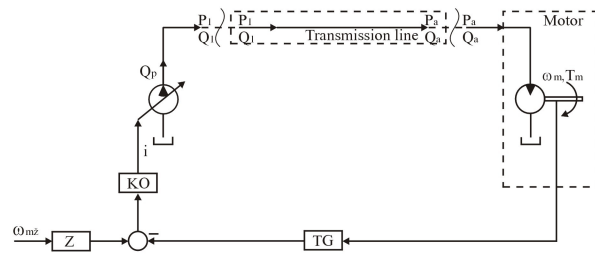


Figure 1. Symbolic diagram of the servo pump controlled motor

### DYNAMICAL MATHEMATICAL MODELS OF THE COUPLED SYSTEM PUMP - TRANSMISSION LINE - MOTOR

The structure diagram of the coupled system pump-transmission line- motor is given in Fig 2

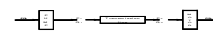


Figure 2. The structure diagram of the coupled subsystem

The fundament of the different developed dynamical models of the coupled system was presented in the [7].

According to the aim of this paper, here are given the transfer functions of the model with distributed parameters and lumped parameters ("T" and " $\pi$ " model).

For the model with distributed parameters, the transfer function has form:

$$\frac{\bar{\omega}_m}{\bar{\omega}_p}(s) = \frac{1}{\left[1 + \frac{Z_T}{R_p} + \frac{Z_T}{R_m}\right] \text{ch}(\Gamma l) + \left[\frac{Z_T}{Z_c} + \frac{Z_c}{R_p} + \frac{Z_c Z_T}{R_p R_m}\right] \text{sh}(\Gamma l)} \quad (1)$$

For the "T" model, the transfer function has form:

$$\frac{\bar{\omega}_m}{\bar{\omega}_p}(s) = \frac{1}{\left[Y + \frac{1}{R_p} \left(1 + \frac{YZ}{2}\right)\right] Z_T + \left[\frac{Z}{R_p} \left(1 + \frac{YZ}{4}\right) + \left(1 + \frac{YZ}{2}\right)\right] \left[1 + \frac{Z_T}{R_m}\right]} \quad (2)$$

For the "π" model, the transfer function has form:

$$\frac{\bar{\omega}_m}{\bar{\omega}_p}(s) = \frac{1}{\left[Y \left(1 + \frac{YZ}{4}\right) + \frac{1}{R_p} \left(1 + \frac{YZ}{2}\right)\right] Z_T + \left[\frac{Z}{R_p} + \left(1 + \frac{YZ}{2}\right)\right] \left[1 + \frac{Z_T}{R_m}\right]} \quad (3)$$

$$\Gamma l = \sqrt{ZY}; Z_c = \sqrt{\frac{Z}{Y}}; Z = R + Ls; Y = Cs; \quad (4)$$

Where:

$$R_T = \frac{B_v}{D_m^2}; L_T = \frac{J_m}{D_m^2}; Z_T = R_T + L_T s$$

$$L = \frac{\rho l}{A}; R = \frac{128 \mu l}{\pi d^4}; C = \frac{Al}{E}; R_{ref} = \frac{P_{ref}}{Q_{ref}} \quad (5)$$

DYNAMICAL MATHEMATICAL MODELS OF THE SERVO PUMP CONTROLLED MOTOR

Based on the previous models and the pump dynamical model [14], including measurement and control parameters is obtained the open loop transfer function of the servo system.

For the servo system with distributed parameters, the transfer function has from:

$$W_{ok} = \frac{K}{\left[\frac{s^2}{\omega_n^2} + \frac{2\xi}{\omega_n} s + 1\right]} \cdot \left\{ \left[1 + \frac{Z_T}{R_p} + \frac{Z_T}{R_m}\right] \text{ch}(\Gamma l) + \left[\frac{Z_T}{Z_c} + \frac{Z_c}{R_p} + \frac{Z_c Z_T}{R_p R_m}\right] \text{sh}(\Gamma l) \right\}$$

For the servo system with "T" line model, the transfer function has from:

$$W_{ok} = \frac{K}{\left[\frac{s^2}{\omega_n^2} + \frac{2 \cdot \xi}{\omega_n} s + 1\right]} \cdot \left\{ \left[ Y^1 + \frac{1}{R_p} \left(1 + \frac{Y^1 Z^1}{2}\right) \right] Z_T + \left[ \frac{Z^1}{R_p} \left(1 + \frac{Y^1 Z^1}{4}\right) + \left(1 + \frac{Y^1 Z^1}{2}\right) \right] \cdot \left[1 + \frac{Z_T}{R_m}\right] \right\}$$

For the servo system with "π" line model, the transfer function is given by:

$$W_{ok} = \frac{K}{\left[\frac{s^2}{\omega_n^2} + \frac{2 \cdot \xi}{\omega_n} s + 1\right]} \cdot \left\{ \left[ Y^1 \left(1 + \frac{Y^1 Z^1}{4}\right) + \frac{1}{R_p} \left(1 + \frac{Y^1 Z^1}{2}\right) \right] Z_T + \left[ \frac{Z^1}{R_p} + \left(1 + \frac{Y^1 Z^1}{2}\right) \right] \left[1 + \frac{Z_T}{R_m}\right] \right\}$$

Where:  $Y^1 = Y \cdot l, Z^1 = Z \cdot l, K = \frac{K_a K_{TG} k_p k_{ii} \omega_p}{D_m}$

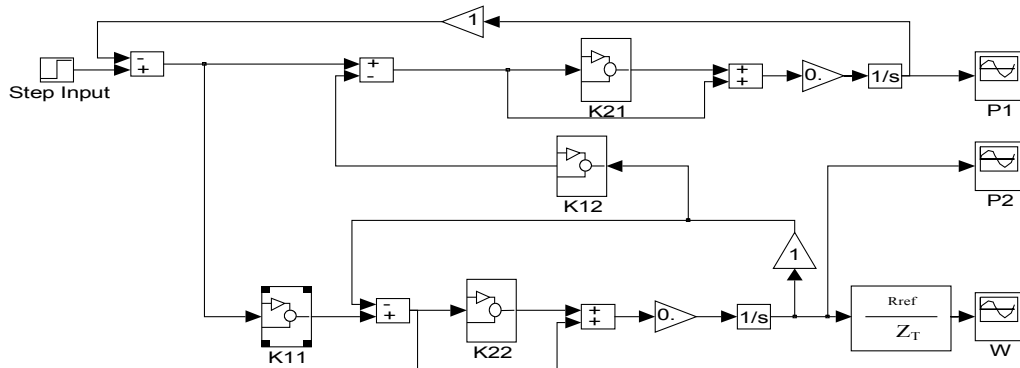


Figure 3. The Simulation model in MATLAB – Simulink

THE SIMULATION ANALYSIS IN FREQUENCY AND TIME DOMEN

For the analysis of dynamic behavior for the servo system authors of this paper developed the simulation modal based on MATLAB - SIMULINK (Fig.3. ). The subsystems K11, K12, K21 and K22 are determined by the model method and given by:

$$K_{21} = K_{22} = \frac{1}{ch(\Gamma l)} = \frac{1}{\alpha} \sum_{i=1}^n \frac{a_i \bar{s} + b_i}{\bar{s}^2 + 2\zeta_i \omega_{ni} \bar{s} + \omega_{ni}^2}$$

$$K_{11} = K_{12} = \frac{Z_c ch(\Gamma l)}{Z_{ca} sh(\Gamma l)} = \frac{1}{D_n \bar{s}} \left[ 1 + \sum_{i=1}^n \frac{\frac{D_n a_i}{Z_{ca}} \bar{s}^2 + \frac{D_n b_i}{Z_{ca}} \bar{s}}{\bar{s}^2 + 2\zeta_i \omega_{ni} \bar{s} + \omega_{ni}^2} \right]$$

The values of the parameters are as follows:

- $E = 1,43 \cdot 10^9 \text{ N/m}^2; \rho = 860 \text{ kg/m}^3; \eta = 0,033 \text{ Ns/m}^2;$
- $R_p = R_m = 1 \cdot 10^{11} \text{ Nm}^{-2} / \text{m}^3 \text{ s}^{-1}; i = 3 \text{ mA}; p_{ref} = 100 \text{ bar};$
- $Q_{ref} = 5 \cdot 10^{-4} \text{ m}^3 / \text{s} = 30 \text{ l/min}; d = 15 \cdot 10^{-3} \text{ m}; l = 10 \text{ m};$
- $c = 1290 \text{ m/s}; D_m = 2.61 \cdot 10^{-5} \text{ m}^3 / \text{rad}; B_v = 0.2 \text{ Nms};$
- $I_m = 2 \text{ kgm}^2; I_m = 0,02 \text{ kgm}^2; \omega_n = 100 \text{ rad/s}; \zeta = 0.628$

The load motor inertia effect on dynamic behavior of the coupled system pump-line-motor is shown in Fig.4. in frequency domen. Domination of the motor inertia on dynamic behavior the coupled system over the domination of the hydraulic line dynamic is obvious only for very high the motor inertia (Fig.4.).

Comparative analysis of the coupled model with distributed parameters and with lumped parameters in frequency and time domen (Fig.5.-Fig.7.) shows that:

- Dynamic of the long coupled system with the transmission line model with distributed parameters is very complex,
- behavior the long coupled system with the transmission line model with lumped parameters is not authentic and
- "π" Transmission line model is more reliable than "T" model.

Comparative analysis of stability of the servo system with and without the transmission line model with distributed parameters (Fig.9.-Fig.10.) in frequency domen shows that the stability conditions of the servo system with the transmission line model with distributed parameters be fulfilled for much more amplification ( $K < 0,25$ ), then for the servo system with transmission line model with lumped parameters ( $K < 20$ ).

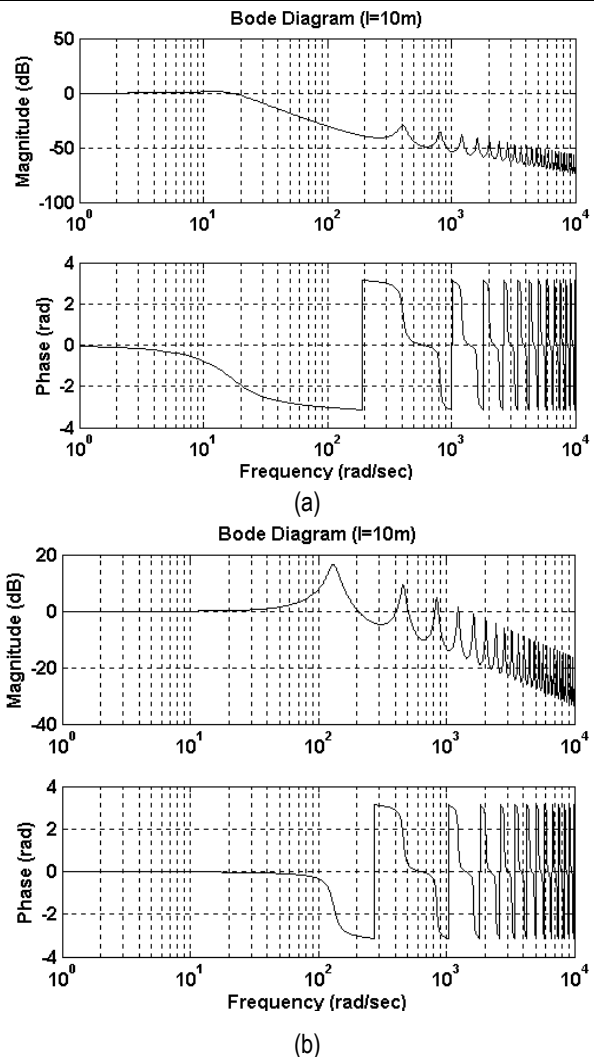


Figure 4. Amplitude and frequency characteristics of the coupled system with distributed parameters (a)  $I_m = 2 \text{ kgm}^2$  (b)  $I_m = 0.02 \text{ kgm}^2$

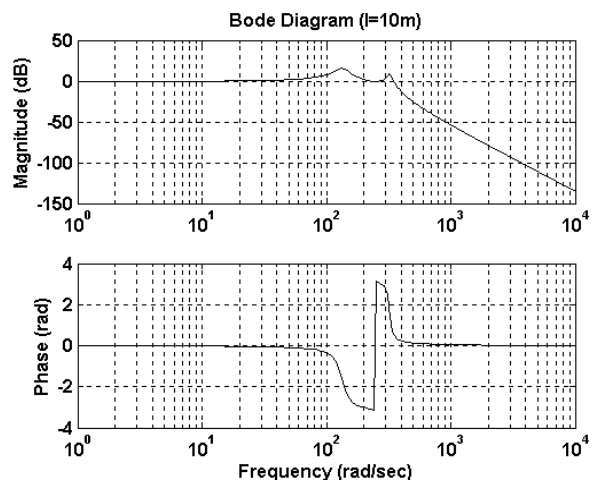


Figure 5. Amplitude and frequency characteristics of the coupled system with fixed parameters "TI"  $I_m = 0.02 \text{ kgm}^2$

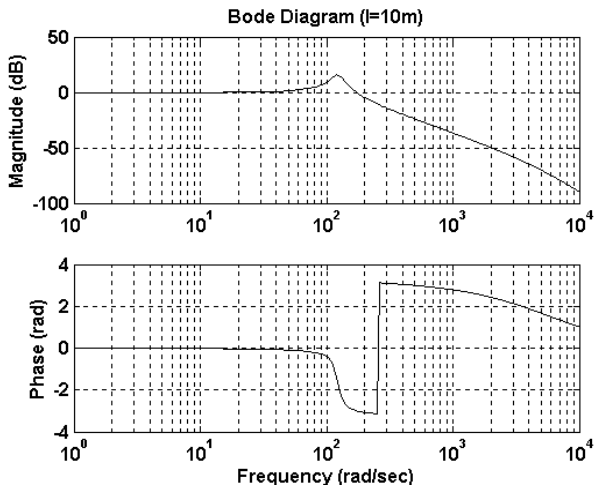


Figure 6. Amplitude and frequency characteristics of the coupled system with fixed parameters  $T''I_m = 0.02 \text{ kgm}^2$

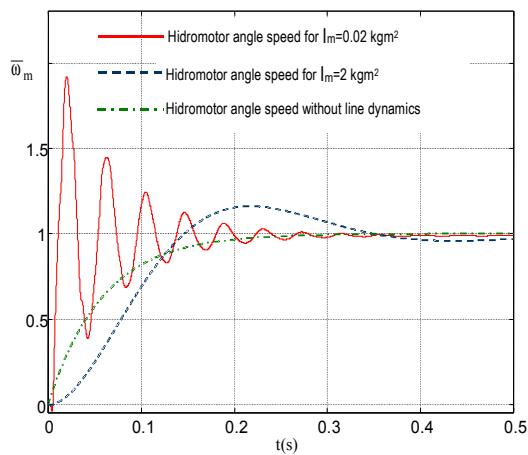


Figure 7. Transient relative motor speed, variation with time

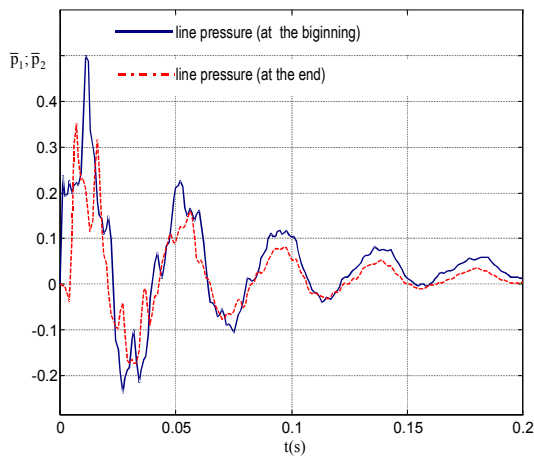
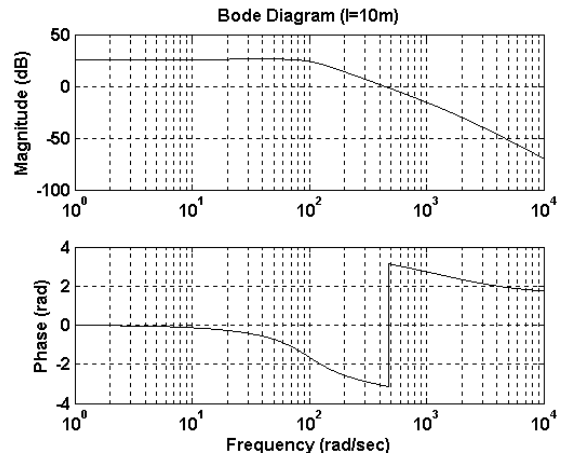
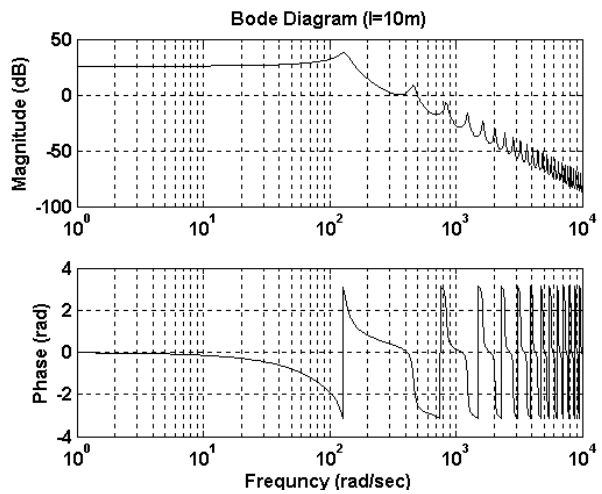


Figure 8. Transient relative line pressure variation with time (at the beginning and the end)



(a)



(b)

Figure 9. Amplitude and frequency characteristics of open-loop servo system with distributed parameters (a) without line dynamics (b) with line dynamics

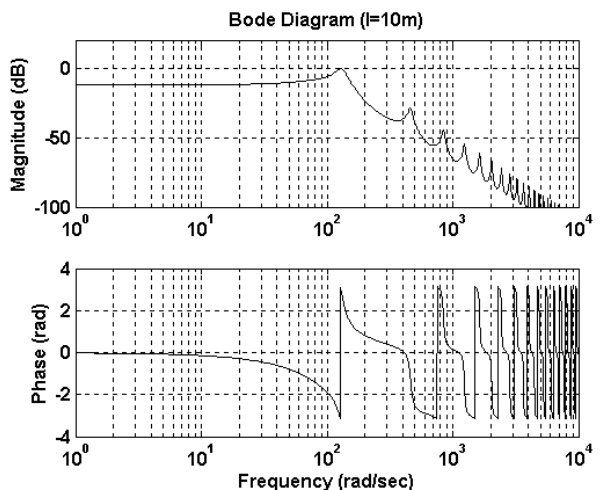


Figure 10. Amplitude and frequency characteristics of open-loop servo system with distributed parameters for  $K=0.25$ .

## CONCLUSIONS

Analysis of dynamic behavior of servo controlled hydrostatic power transmission system with long transmission line given in this paper shows:

- The long transmission line model with distributed parameters has significant effect on dynamic behavior of the servo systems.
- The stability conditions are much more rigorous for the servo system with long transmission line than for the servo system with short transmission line (this is obvious for  $l > 10$  m).
- A good servo system design demands are that dynamics of the components and the coupled system pump- transmission line – motor must be included.

## NOTATION

A – rod cross-sectional-area  
 $B_v$  – friction viscosity  
 C – line capacitance  
 d- line diameter  
 $D_m$  – displacement motor  
 $D_p$  – displacement pump  
 $D_n$  – dissipation number  
 E – modulus of elasticity  
 $K_a$  – coefficient current-voltage  
 $k_{1i}$  – coefficient of current application  
 $J_m$  – momentum inertia  
 l – line length  
 n – number of modes  
 $P_1, P_2$  – line pressure at inlet and outlet  
 $P_{ref}$  – simulation reference pressure  
 $Q_{ref}$  – reference flow rate  
 R – fluid resistance  
 $R_{ref}$  – simulation reference resistance  
 $R_p$  – leakage pump resistance  
 $R_m$  – leakage motor resistance  
 s- Laplace operator  
 $Q_1, Q_2$  – line flow rates at inlet and outlet  
 Z – series impedance  
 $Z_c$  – characteristic impedance  
 $Z_T$  – motor impedance  
 Y – shunt admittance  
 $\Gamma$  – Propagation operator  
 $\rho$  – Fluid density

$\mu$  – Fluid viscosity  
 $\xi$  – Damping ratio  
 $\omega_n$  – natural frequency  
 $\bar{\omega}_m$  – non dimensional motor angular speed  
 $\bar{\omega}_p$  – non-dimensional pump angular speed

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