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TOPOGRAPHICAL METHOD FOR SOLVING PROBLEMS OF THREE-PHASE CIRCUITS

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ABSTRACT
A topographical method of drawing of alternating current circuits used for analysis and synthesis of three-phase circuits for determination of phase sequence is treated. The main point of the method is drawing of circuit diagram and all its connections and elements on a complex plane. Every node of the circuit has coordinates, which determine a complex number equal to the voltage of the node in a complex form. A simple diagram for determination of phase sequence with a negative-glow lamp is synthesised.

INTRODUCTION

The main methods of analyse of alternating current circuit is the symbolical method. The current and voltage, which are sinusoidal quantities and are time functions, are noted as complex numbers $Ae^{j\varphi}$ where A is the module of the complex number and is equal to the current or voltage, angle φ is the phase angle and j is the imaginary unit. The impedance is noted also as a complex number $Z = ze^{j\varphi}$. This method allows to use directly the laws and methods of calculation of direct current circuits and all calculations are made by complex numbers. If he expressions are complicated, the main difficulty is calculative, because it is necessary to convert the complex numbers in algebraic form when adding and subtracting are made and in index form when multiplication and division are made. This difficulty drop of when use calculator that could calculate complex expressions (for example TI 89). This calculator is able to calculate random complicated expressions and complex numbers could be inputted both in algebraic and index form.

As the complex numbers are represented as vectors in a complex plane, all currents and voltages could be represented as vectors. The vector diagram gives a clear idea for current state in determined values of impedance and frequency.

If we are interested in a particular vector - current or voltage and its changes as a result of change of particular element of the circuit or frequency, we should find the vector hodograph, i.e. the geometrical locus (GL) that the vertex of the vector covers in the whole diapason of change of the variable parameter.

A topographical method [3] is worked out, in which the electrical diagram is not driving in standard way but is transferred on complex plane. Every node takes defined place which coordinates that determine a complex number equal to the voltage of this node. The nodes are divided in two groups: static and mobile. In static nodes the voltage does not depend on diagram elements. Such are the clamp of electromotive force source. All the rest elements are mobile, i.e. the voltage on them is changing.

For illustration (fig.1) is selected a RC group, connected to the voltage source U_{12} in two ways:

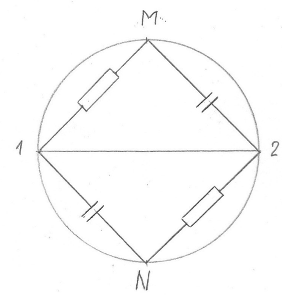

Figure 1.

Nodes 1 and 2 are static – they are connected to the voltage source. In first way resistor is connected to p.1 and capacitor to p.2. The mobile node M moves on the upper semi-circumference when the value of element or frequency is changed. That is so because the voltages U_{1M} and U_{2M} , which correspond to the segments M1 and M2 are on 90^0 one towards another. When connect the resistor to the p.2 and capacitor to p.1, the mobile node N moves on the bottom seim-circumference.

ANALYSIS OF THE CIRCUIT IN ORDER TO DETERMINE THE PHASE SEQUENCE

When two lamps and a capacitor are connected in star connection the popular diagram for determination of phase sequence is obtained. [2]. This circuit was drawn on fig. 2 by the described topographical way.

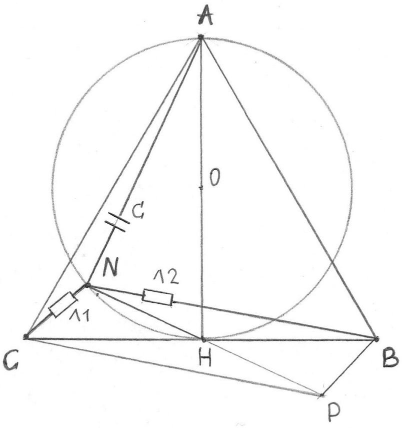

Figure 2.

Points A, B and C are static nodes, connected to the supplying three-phase voltage. The two lamps L_1 and L_2 are identical and have the same conductance Y. In determined scale (conditionally Y=1) vectors of the current coincidence with vectors of the voltage, because the lamp often have pure active resistance. $\overline{NC}$ - vector of current and voltage for lamp L_1. $\overline{NB}$ - vector of current and voltage for lamp L_2. The sum of the two currents through the lamps L_1 and L_2 coincide with the vector $\overline{NP}$.

$$\overline{NP} = \overline{NC} + \overline{NB} \qquad (1)$$

NP is the diagonal in the parallelogram NBPC and passes through the middle of CB, i.e. $\overline{NP} = 2\overline{NH}$. Or:

$$Y.U_{NC} + Y.U_{NB} = Y.U_{NP} = 2Y.U_{NH} \qquad (2)$$

Expression (2) allows to change both lamps L_1 and L_2 with active resistor R_{NH} , connected between p.N and p.H, which has two times higher conductance and is equal to 2Y. This way the three-element diagram (star connection) becomes two-element one - resistor R_{NH} is series connected to the capacitor C. In this type of connection mobile node N moves on the left semi-circumference which diameter is AH. This conclusion is made in [1] using the analytical expressions of the circle diagram theory.

We consider the problem only geometrically and in this way it is possible to present better the motion of node N as well as to make a mechanical analogue. Two lamps and a capacitor are presented as springs, one end of which is attached respectively to points A, B и C, and another end to the mobile point N. The spring stiffness к (f = k.x) corresponds exactly to the conductance Y of the element. Point N is attached on the periphery of the rotating round the point O circle with diameter AH. So p. N of this mechanical model will stop on the place, exactly corresponding to the voltage of the node N in complex plane. The distances conform to the voltages and amperage, i.e. expressions $I = Y.U$ and $f = k.x$ are analogous. For example, if the capacitor capacity increases, the spring stiffness AN increases too. However the circle will turn to the clockwise and p. N will come near the p. A. The voltage on the capacitor will decrease, and voltage on the lamps – will increase.

In positive phase sequence the lamp L_2 shines brighter because NB >NC. The lamps are for nominal voltage 230V. If p. N takes away from p. A, the voltage on lamp L_2 will be higher than nominal voltage and the lamp will be destroyed. In the expression, solved in [2] is sat that conductance of lamps and capacitor are equal. So the voltage on lamp L_2 is 330V and it will be destroyed.

Geometrically we could quickly determine the conductor capacity, in which the voltage on the lamps is secure. In corresponding scale we determine the segment NB, and this segment corresponds to the voltage 230V (segment CB corresponds to 380V). We draw the circle with centre p.B that crosses the circle on fig.2 in p. N. We should write:

$$\frac{Y_C}{Y_{NH}} = \frac{Y_C}{2Y} = \frac{NH}{AN} ,$$

$$\omega C = Y_C 2Y \frac{NH}{AN}$$

$$C = \frac{2Y}{\omega} . \frac{NH}{AN} \qquad (3)$$

Expression (3) determines the capacitor capacity when we have chosen two identical lamps with conductance Y.

SYNTHESIS OF THE CIRCUIT IN ORDER TO DETERMINE THE PHASE SEQUENCE

The most important advantage of the topographical drawing of the electrical circuit of alternating current is the opportunity to create a new diagram, which working could be presented graphically.

In this report we will consider a simple circuit synthesised in order to determine the phase sequence. In described above circuit, one lamp shines brighter than another. This indication is not enough clear. The phase determination circuit should have only one lamp, which is on in positive phase sequence and is off in negative phase sequence.

The circuit consists of tow resistors R, two capacitors C, a negative-glow lamp L for 220V and limit resistor $r = 360k\Omega$ (fig. 3a). The impedance of the resistors and capacitors is identical, i.e. $R = \frac{1}{\omega C}$. On fig. 3b the circuit is drown topographically.

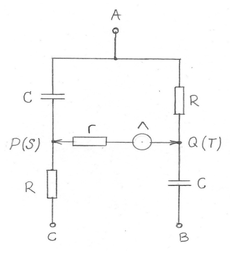

Figure 3a

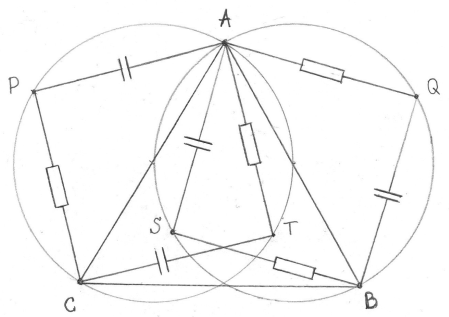

Figure 3b

On the side AC of the triangle ABC is drown the circle with diameter AC. On the side AB is drown another circuit. Mobile node P is the vertex of the rectangular isosceles triangle ACP that lies on the circle. This way we find the mobile node Q on the second circle. Linear voltage $U_Л = 380V$ corresponds to the sector m and:

m=AB=BC=CA

$$\angle PAQ = 45^0 + 60^0 + 45^0 = 150^0$$

$$PA = QA = \frac{m}{\sqrt{2}} = 0,7071m \quad (4)$$

$$PQ = 2PA\sin\frac{\angle PAQ}{2} = 2\frac{m}{\sqrt{2}}\sin 75^0 = 1,366m \quad (5)$$

From (5) we could determine the voltage between mobile nodes P and Q:

$$U_{PQ} = 1,366.380 = 519,1V \quad (6)$$

In the method of connection, shown on fig.3a, the voltage on the negative-glow lamp is $U_{PQ} = 519V$. The negative-glow lamp is on and it is indication for the right phase sequence.

If we change the places of phases B and C as a result is the system with negative phase sequence. Mobile node T from RC group connected between p.A and p.C lies on the left circle and is the vertex of isosceles triangle ACT. Another mobile node S lies on the second circle and is a vertex of isosceles triangle ABS. Than the voltage on the negative-glow lamp is U_{ST}, and is determined by fig. 3b. $\angle SAT = \angle SAQ - \angle TAQ$. Angle $\angle SAQ$ is an internal angle of the square AQBS and is equal to 90°. Angle $\angle TAQ$ is equal to 60^0 because the right circle together with inscribed in it square is obtained from the left by 60^0 rotation round p. A. As a result of this rotation the diameter AC comes over AB, and side AT over AQ.

$$\angle SAT = 90^0 - 60^0 = 30^0$$

$$ST = 2SA\sin\frac{\angle SAT}{2} = 2\frac{m}{\sqrt{2}}\sin 15^0 = 0,3660m \quad (7)$$

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The segment ST corresponds to the voltage on the negative-glow lamp. From (7) we determine the voltage between mobile nodes S and T:

$$U_{ST} = 0,3660.380 = 139,1V \quad (8)$$

This voltage is applied on the negative glow lamp when the phase sequence is negative. The lamp is off because the its starting voltage is 170V. Two resistors and two capacitors work under the same voltage. We determine this voltage by using of (4):

$$U_{PA} = 0,7071.380 = 268,7V \quad (9)$$

For resistor s R we have selected $R = 33k\Omega$; 2W. The power on them is $P = 2,188W$, and there are no problems when in short time we turn it on many times.

The capacitor capacity is $C = \frac{1}{\omega R} = \frac{1}{100\pi\, 33.10^3} = 96,46nF$. The capacitor is polyester one and has the nominal value $C = 100nF, U = 400V$.

Dimensions of the plate together with all elements are 3X5 cm. The three terminals are numbered - 1,2 and 3, and when they are connected to the system with positive phase sequence the lamp is on and to the system with negative phase sequence the lamp is off.

CONCLUSION

The diagram shown on fig.3a is realised and tested in three-phase systems with linear voltage 127V, 220V and 380V. The diagram is used for determination of phase sequence. The voltages U_{PQ} and U_{ST} are measured by using of the high-resistance voltmeter (10MΩ) when the negative-glow lamp is connected or is not connected. The voltage U_{PQ} decreases about 5%, and voltage U_{ST} is constant.

Presented topographical method of drawing of an alternating current circuit and the mechanical model, created on this base gives the new opportunities to understand these diagrams and to synthesise new circuits with particular applications.

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