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RADIATIVE HEAT TRANSFER EQUATION IN SYSTEMS OF GREY - DIFFUSE SURFACES SEPARATED BY NON-PARTICIPATING MEDIA

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ABSTRACT
The paper proposes a matrix model for solving the radiative heat transfer in enclosures based on mutual angular radiation coefficients and radiative properties of the surfaces that make up the enclosure.

INTRODUCTION

Radiative heat transfer represents the most important part of the global heat exchange in thermal installations working at temperatures, such as steam generators, boilers, furnaces and so on. The two components of the radiative heat transfer – (1) between two or more surfaces and (2) between gas and surfaces – on the one hand, and on the other hand the complex geometry of most heat exchange systems, make the simulation of the radiative heat transfer a complex process, more suited for the numerical approach. Besides, a serious drawback is the lack of a standard terminology in the field of radiative heat transfer. The nomenclature used in this paper is based on that of the illumination engineering community due to the close relationship between physical terms and standardized terms used in the field of illumination engineering.

The basic concepts of radiative heat transfer such as direction and solid angle, radiant intensity, radiosity, emittance, absorbance, reflectance and so on, are considered known and aren't discussed in this paper. The basic geometrical parameters are represented in figure 1. A summary of the basic terms used henceforth is given below:
- spectral radiation intensity due to the own emission of the surface:

$$I_{\lambda,e} = \frac{\delta \dot{Q}}{\cos\varphi \; dS_1 d\Omega \; d\lambda}$$

Index *e* means self-emitted.
- spectral power density:

$$E_\lambda(\lambda) = \int_0^{2\pi} d\psi \int_0^{\pi/2} I_{\lambda,e}(\lambda,\varphi,\psi)\cos\varphi \; \sin\varphi \, d\varphi$$

- spectral radiation intensity due to incident radiation:

$$I_{\lambda,i} = \frac{\delta \dot{Q}}{\cos\varphi \; dS_1 d\Omega \; d\lambda}$$

Index i means incident to the surface considered.
- spectral irradiation:

$$G_\lambda(\lambda) = \int_0^{2\pi} d\psi \int_0^{\pi/2} I_{\lambda,i}(\lambda,\varphi,\psi)\cos\varphi \; \sin\varphi \; d\varphi$$

spectral radiosity:

$$J_\lambda(\lambda) = \int_0^{2\pi} d\psi \int_0^{\pi/2} I_{\lambda,e+r}(\lambda,\varphi,\psi)\cos\varphi \; \sin\varphi \; d\varphi$$

Radiosity takes into account both self-emitted and reflected radiation

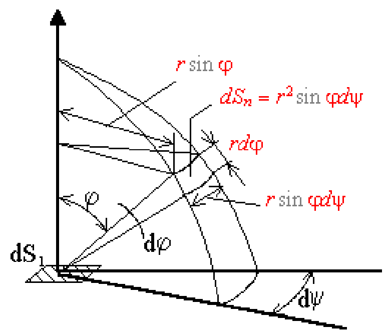

Figure 1

RADIATIVE HEAT TRANSFER IN A SYSTEM OF SURFACES

The system of surfaces considered consists of n flat diffuse-grey surfaces which make up an enclosure.
The net radiative heat flux of a surface i is given by the difference between the radiated heat flux due to emission and reflection and the incident radiative heat flux between two random patches (figure 2):

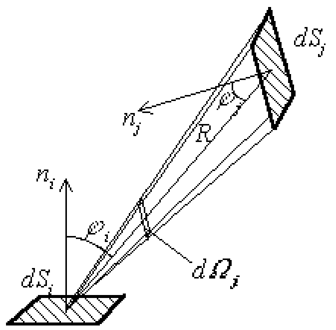

Figure 2

$$\dot{Q}_i = S_i(J_i - G_i) \tag{1}$$

in which the radiosity of surface *i* is given by: $J_i = E_i + R_i G_i$ with R_i – reflectance of the surface *i*; for an opaque surface, $R_i = 1 - A_i$, and (1) turns into:

$$Q_i = S_i(E_i - A_i G_i) \tag{2}$$

The radiative heat flux given by (2) has a non-zero value unless the considered surface gets an equivalent heat flux either through convection or conduction. If heat exchange is accomplished exclusively through radiation and if steady state is reached, then $\dot{Q}_i = 0$.

For diffuse-grey surfaces the emittance equals the absorbtance according to Kirchhoff's law: $\varepsilon_i = A_i$. In this case the radiosity is given by:

$$J_i = \varepsilon_i E_{b,i} + (1 - A_i)G_i \tag{3}$$

with $E_{0,i}$ the power density of the black body having the same temperature as the surface considered.
The above equation changes (1) to:

$$\dot{Q}_i = S_i\left(J_i - \frac{J_i - \varepsilon_i E_{b,i}}{1 - \varepsilon_i} \right) \tag{4}$$

or:

$$\dot{Q}_i = \frac{E_{b,i} - J_i}{\dfrac{1 - \varepsilon_i}{\varepsilon_i S_i}} \tag{5}$$

Equation (5) describes the radiative heat exchange from a surface. It suggests an analogy with Ohm's law leading to the definition of the radiative resistance of the surface *i*:

$$R_{r,i} = \frac{1 - \varepsilon_i}{\varepsilon_i S_i} \tag{6}$$

If the surface *i* belongs to an enclosure then the radiosity J_i depends on the heat transfer among surface *i* and surfaces *j* (*j*=1...*n*). The radiative heat flux radiated by surface *i* and received by the surface *j* is given by:

$$Q_{i\to j} = \varphi_{ij} J_i S_i \tag{7}$$

and the radiative heat flux radiated by surface *j* and received by the surface *i* is given by:

$$Q_{j\to i} = \varphi_{ji} J_j S_j \tag{8}$$

In which φ_{ij} and φ_{ji} are the mutual medium angular radiation coefficients.

Considering a system consisting of two surfaces *i* and *j*, the mutual medium angular radiation coefficient φ_{ij} is defined as the ratio between the radiative heat flux emerging from S_i and intercepted by S_j and the overall radiative heat flux emerging from S_i:

$$\varphi_{ij} = \frac{\dot{Q}_{i\to j}}{J_i S_i} \tag{9}$$

Isolating two elementary patches dS_i and dS_j and applying Lambert's law, the elementary radiative heat flux emerging from dS_i in direction φ_i in the elementary solid angle $d\Omega_{ji}$ is given by:

$$\delta \dot{Q}_{i\to j} = I_i \cos\varphi_i \, dS_i \, d\Omega_{ji} \tag{10}$$

Taking into account the definition of the solid angle, the above expression turns into:

$$\delta \dot{Q}_{i\to j} = I_i \frac{\cos\varphi_i \cos\varphi_j}{R^2} dS_i dS_j \tag{11}$$

For diffuse surfaces, $I_i = \frac{J_i}{\pi}$ and equation (11) changes to the following form:

$$\delta \dot{Q}_{i\to j} = J_i \frac{\cos\varphi_i \cos\varphi_j}{\pi R^2} dS_i dS_j \tag{12}$$

As a result of integration, equation (12) turns into:

$$Q_{i\to j} = J_i \int_{S_i}\int_{S_j} \frac{\cos\varphi_i \cos\varphi_j}{R^2} dS_i dS_j \tag{13}$$

The net radiative heat flux between surfaces I and j is given by:

$$\dot{Q}_{ij} = \dot{Q}_{i\to j} - \dot{Q}_{j\to i} \tag{14}$$

Taking into account the well-known reciprocity property of the medium angular radiation coefficients (14) becomes:

$$Q_{ij} = \varphi_{ij} S_i (J_i - J_j) \tag{15}$$

or:

$$\dot{Q}_{ij} = \frac{J_i - J_j}{R_{r,ij}} \qquad (16)$$

In which the geometric radiative resistance $R_{r,ij}$ is given by:

$$R_{r,ij} = \frac{1}{\varphi_{ij} S_i} = \frac{1}{S_{ij}} \qquad (17)$$

The geometric radiative resistance $R_{r,ij}$ depends solely on the geometry of the radiant system, unlike $R_{r,i}$, which depends also on the radiative properties of the material.
The total incident heat flux on the surface i is given by:

$$G_i S_i = \sum_{j=1}^{n} \varphi_{ji} S_j J_j = \sum_{j=1}^{n} \varphi_{ij} S_i J_j \qquad (18)$$

from which emerges the irradiation of surface *i*:

$$G_i = \sum_{j=1}^{n} \varphi_{ij} J_j \qquad (19)$$

Combining (1) and (19):

$$\dot{Q} = S_i \left(J_i - \sum_{j=1}^{n} \varphi_{ij} J_j \right) \qquad (20)$$

or, taking into account the enclosure property of the medium angular radiation coefficients $J_i S_i = \sum_{j=1}^{n} \varphi_{ij} J_i S_i$, equation (20) yields consecutively:

$$Q_i = \sum_{j=1}^{n} S_i \varphi_{ij} J_i - S_i \sum_{j=1}^{n} \varphi_{ij} J_j \qquad (21)$$

$$\dot{Q}_i = \sum_{j=1}^{n} S_i \varphi_{ij} \left(J_i - J_j \right) = \sum_{j=1}^{n} \dot{Q}_{ij} \qquad (22)$$

Equation (5) and (22) lead to:

$$\frac{E_{b,i} - J_i}{\dfrac{1-\varepsilon_i}{\varepsilon_i S_i}} = \sum_{j=1}^{n} \frac{J_i - J_j}{\dfrac{1}{\varphi_{ij} S_i}} \qquad (23)$$

In a system of grey-diffuse surfaces (23) is written as:

$$\frac{E_i - J_i}{R_i} = \sum_{j=1}^{n} \frac{J_i - J_j}{R_{ij}} \qquad (24)$$

or:

$$\frac{E_i}{R_i} - \frac{1}{R_i} J_i = J_i \sum_{j=1}^{n} \frac{1}{R_{ij}} - \sum_{j=1}^{n} \frac{1}{R_{ij}} J_j \qquad (25)$$

which, upon rearranging, becomes:

$$\frac{E_i}{R_i} + \sum_{j=1}^{n} \frac{1}{R_{ij}} J_j = \frac{1}{R_i} + \sum_{j=1}^{n} \frac{1}{R_{ij}} J_i \qquad (26)$$

The radiosity is given by:

$$J_i = \frac{\dfrac{1}{R_i}}{\dfrac{1}{R_i} + \sum_{j=1}^{n} \dfrac{1}{R_{ij}}} E_i + \frac{1}{\dfrac{1}{R_i} + \sum_{j=1}^{n} \dfrac{1}{R_{ij}}} \sum_{j=1}^{n} \frac{1}{R_{ij}} J_j \qquad (27)$$

or:

$$\begin{bmatrix} J_1 \\ J_2 \\ .. \\ .. \\ J_n \end{bmatrix} = \begin{bmatrix} \dfrac{\rho_1}{\Phi_1} E_1 \\ \dfrac{\rho_2}{\Phi_2} E_2 \\ .. \\ .. \\ \dfrac{\rho_n}{\Phi_n} E_n \end{bmatrix} + \begin{bmatrix} \Phi_1 \rho_{11} & \Phi_1 \rho_{12} & \cdots & \Phi_1 \rho_{1n} \\ \Phi_2 \rho_{21} & \Phi_2 \rho_{22} & \cdots & \Phi_2 \rho_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \Phi_n \rho_{n1} & \Phi_n \rho_{n2} & \cdots & \Phi_n \rho_{nn} \end{bmatrix} \bullet \begin{bmatrix} J_1 \\ J_2 \\ .. \\ .. \\ J_n \end{bmatrix} \qquad (28)$$

in which: $\Phi_i = \frac{1}{R_i} + \sum_{j=1}^{n} \frac{1}{R_{ij}}$, $\rho_i = \frac{1}{R_i}$ and $\rho_{ij} = \frac{1}{R_{ij}}$.

Subtracting from both left and right member the term representing the radiosities matrix, equation (28) turns into:

$$\begin{bmatrix} 0 \\ 0 \\ .. \\ .. \\ 0 \end{bmatrix} = \begin{bmatrix} \dfrac{\rho_1}{\Phi_1} E_1 \\ \dfrac{\rho_2}{\Phi_2} E_2 \\ .. \\ .. \\ \dfrac{\rho_n}{\Phi_n} E_n \end{bmatrix} + \left(\begin{bmatrix} \Phi_1 \rho_{11} & \Phi_1 \rho_{12} & \cdots & \Phi_1 \rho_{1n} \\ \Phi_2 \rho_{21} & \Phi_2 \rho_{22} & \cdots & \Phi_2 \rho_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \Phi_n \rho_{n1} & \Phi_n \rho_{n2} & \cdots & \Phi_n \rho_{nn} \end{bmatrix} - I \right) \bullet \begin{bmatrix} J_1 \\ J_2 \\ .. \\ .. \\ J_n \end{bmatrix} \qquad (29)$$

in which I is the identity matrix:

$$I = \begin{bmatrix} 1 & 0 & ... & 0 \\ 0 & 1 & ... & 0 \\ 0 & 0 & ... & 0 \\ 0 & 0 & ... & 1 \end{bmatrix}$$

Finally, the radiosities matrix is the solution of the following system:

$$\begin{bmatrix} 1-\Phi_1\rho_{11} & -\Phi_1\rho_{12} & ... & -\Phi_1\rho_{1n} \\ -\Phi_2\rho_{21} & 1-\Phi_2\rho_{22} & ... & -\Phi_2\rho_{2n} \\ ... & ... & ... & ... \\ -\Phi_n\rho_{n1} & -\Phi_n\rho_{n2} & ... & 1-\Phi_n\rho_{nn} \end{bmatrix} \begin{bmatrix} J_1 \\ J_2 \\ ... \\ J_n \end{bmatrix} = \begin{bmatrix} \frac{\rho_1}{\Phi_1}E_1 \\ \frac{\rho_2}{\Phi_2}E_2 \\ ... \\ \frac{\rho_n}{\Phi_n}E_n \end{bmatrix} \quad (30)$$

CONCLUSIONS

Equation (30) can be solved by mean of numerical methods such Gauss-Siedel iteration. Gauss-Siedel iteration has the advantage of being absolutely convergent for diagonally dominant systems such as the one of interest here. A matrix *M* is diagonally dominant if for all *i* $\sum_{j=1, j\neq i} |M_{ij}| < |M_{ii}|$. This is the case of the matrix in equation (30) because medium angular radiation coefficient from a flat surface to itself is 0.

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