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ALPHA STABLE GEOSTATISTICAL MODEL IN MINERAL RESOURCES EVALUTION

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ABSTRACT
The author offer Alfa-stable geostatistical model providing the answers to problems: what accuracy of an estimate of reserve of mineral raw material, with what can appear increase of extracted economically cost effective reserve and so on. The model is geostatistics imitative and is founded on the empirical characteristic of allocation of contents of extracted builders in deposits. From many possible probability models of the separate object most representative are the Alpha-stable probability distribution. These distributions have asymmetry and very wide right tail, i.e. they are successful replacement of lognormal distribution. It is known, that lognormal distribution, while, and is unique, on the basis of which the geostatistical theory designed. The offered model consists of some main sub models. The variogramm sub model is clone to correlation function of symmetric Stable random process. 3D kriging sub model designed on the basis of optimization of estimates on values alpha, which is obtained from input data set through a modified method of Press. If the outcomes satisfy the requirement, it is possible will be connected: to a sub model of efficiency of prospecting drilling, to a sub model of economy of prospecting operations, to a sub model of the market of the capitals and others, bound with the concrete tasks. On the basis of offered the theory designs a software environment and a mining deposits are treated given. The obtained outcomes are widely made comments in a context of accuracy and reliability of the obtained estimates. The preliminary outcomes allow assuming, that the offered Alfa-stable geostatistical model is promising improvement of a number of geostatistical models. In a major degree it is possible to consider this model empirically justified. The value of model is possibility combination of our model with expert estimates with the purposes of creation of more objective prognoses of expected increases of mineral resources in unexplored objects.

INTRODUCTION

Zolotarev in this monograph on stable laws developed [44] a method for stochastic modeling further referred as Model with Point Sources of Influence (MPSI). There also he discussed various possible applications of the MPSI in finance, astrophysics, hydrodynamics, etc. [44]. The method is very suitable for modeling chaos medium (cf. [17]).

In this paper we present a new approach in which, a stochastic medium characterized by a Point Sources of Influence with a Poissonian density. The assumption of Poissonian distribution is quite asset returns common in the field of finance, ore field geology [3], [10], [7], [9], hydrogeology [28]. In these areas, the study of various real phenomena involves sampling and measuring of their properties, e.g. asset return volatility. The most common characteristic of these data sets is the large values of sample variation as well as the unimodal-asymmetric shape of the probability distribution (i.e. a small part of the sample data values has very high values, while the large part is characterized with very low values). This phenomenon was extensively analysed by Mandelbrot [22] and Mittnik and Rachev [25], [26], [27], Bakardjiev [3]

It is generally accepted that the choice of the probability distribution functions (pdf's) should be based on the principles and the hypotheses through which the real data sets are described. The most popular hypothesis is the so-called Law of the Proportional Action, which leads to a lognormal distribution of the sample data [1]. This hypothesis is almost canonized. Indeed the log-transformation reduces significantly the variation and changes the shape of the sample distribution, so it looks like normal distribution. Unfortunately in most cases, especially in geological data, the standard χ^2-analysis for the first and second order errors rejects the lognormal assumption.

Mandelbrot [22] showed that an acceptable alternative is the **Stable** (Paretian) probability distributions. These distributions are asymmetric and possess heavy tail. Unfortunately, most of the stable probability distributions (with few exceptions) have no analytical representation. The increments of the process are also α-stable: $x_{t+\tau} - x_t \overset{d}{=} \gamma(f(.+\tau) - f(.))\zeta_1$, and this contradicts with all existing parametric Kriging procedures. Moreover, it is not guaranteed that the mean exists, while the variation is always unbounded. Also, the difference between the stable and lognormal distributions is detectable only for significant number of sample observations (greater than 10000). These are the main reasons that the stable distributions are not very popular for processing of real data, see the discussion in [25] and [26].

The monographs of Zolotarev [44] and Samorodnitsky and Taqqu [35] increased the interest to stable laws for stochastic modeling of real-nature processes (for financial modeling see also [34]).

In this paper we present some promising numerical applications of the MPSI. The obtained numerical results seem to describe a very good approximation of the stochastic behavior mainly observed in real processes.

BRIEF REVIEW ON STABLE LAWS

The stable distributions were introduced by Paul Levi [21].

By definition, a univariate distribution function $F(x)$ is stable if and only if its characteristic function has the form

$$\varphi(t) = \int e^{itx} dF(x)$$
$$\varphi(t) = \exp\left(iat - \gamma|t|^{a}[1 - i\beta\, sign(t)\omega(t,a)]\right),$$

where

$$\omega(t,a) = \begin{cases} \tan\frac{a\pi}{2} & if a \neq 1 \\ \frac{2}{\pi}\ln|t| & if a = 1 \end{cases},$$

$$sign(t) = \begin{cases} 1, & if t > 0 \\ 0, & if t = 0 \\ -1, & if t < 0 \end{cases},$$

And
$$0 < \alpha \leq 2, -1 \leq \beta \leq 1, \gamma > 0, -\infty < \alpha < \infty$$

The stable distribution is completely determined by four parameters α, β, γ and α where:

α is called the characteristic exponent. It measures the "thickness" of the tails of α-stable distribution. The smaller the value of α, the higher the probability in the distribution tails.

β is a symmetry parameter. The distribution is symmetric about α if $\beta=0$ and is called symmetric α-stable ($S\alpha S$). The Gaussian (α =2 any β) and Cauchy (α =1; $\beta=0$) distributions are both $S\alpha S$.

γ is a scale parameter, also called the dispersion. It is similar to the variance of the Gaussian distribution. However for the Gaussian case (α =2 any β) where σ^2 is the variance.

α is a location parameter. For $S\alpha S$ distributions, it defines the mean if $\alpha \in (1,2]$ and the median if $\alpha \in (0,1]$.

It is simple to be shown that if a random variable X is $S\alpha S$, the characteristic function is of the form

$$\varphi(t) = \exp\left(iat - \gamma|t|^{\alpha}\right).$$

A α-stable distribution is called standard if $\alpha=0$ and $\gamma=1$. If X is a stable random variable with parameters α, β, γ and α, then $(X-\alpha)\gamma^{-\frac{1}{\alpha}}$ is a standardized variable with characteristic component α and symmetry parameter β. In this case, the characteristic function is further simplified to

$$\varphi(t) = \exp\left(-\gamma|t|^{\alpha}\right).$$

GENERALIZED CENTRAL LIMIT THEOREM

According to generalized central limit theorem, the random variable X is the limit in the distribution of normalized sums of the form

$$S_n = (X_1 + \ldots + X_n)/a_n - b_n$$

where $X_1, X_2, \ldots, X_n$ are i.i.d. and if and only if $n \to \infty$ the distribution of X is stable. If the X_i's have finite variance, then the limit distribution is Gaussian. However, if X_i's are with or without finite variance, then the limit distribution is α-stable.

THE MODEL OF POINT SOURCES OF INFLUENCE (MPSI)

From a modeling point of view, the MPSI may be viewed as an analysis of the interactions in a Poissonian ensemble of random shocks, see [44]. The Poissonian ensemble (PE) of points is defined by a random countable system of points in the area $V \subset \Re^n$. Suppose V_1 and V_2 are disjoint sets in V with finite volumes denoted by $|V_1|$ and $|V_2|$ and satisfying the following conditions:

The number of the points in the areas V_1 and V_2 (N_1 and N_2, respectively), are independent random variables.

The probability $P(N_1 = k)$ for k=0,1,2... depends on k and $|V_1|$, but not on the shape of V_1.

If the volume of V_1 decreases to zero, the probability for two or more points in V_1 is negligible in comparison with the probability for exactly 1 point in V_1 that is,

$$P(N_1 = 0) = 1 - \rho|V_1| + o(|V_1|);$$
$$P(N_1 = 1) = \rho|V_1| + o(|V_1|);$$
$$P(N_1 \geq 2) = o(|V_1|);$$

where $\rho\ (\rho > 0)$ is a constant defined as a mean density of the points in the area. It is indeed well known, see for example [44]) that the random function N_1 has Poissonian distribution:

$$P(N_1 = k) = \frac{\lambda^k \exp(-\lambda)}{k!},$$

where the parameter (the intensity) $\lambda = \rho|V_1|$. For different set of points, ρ can be different. The intensity λ can be expressed by the mean number of points in the area and therefore λ is a measure; $\lambda(V_1 + V_2) = \lambda(V_1) + \lambda(V_2)$. In this case, ρ is the density of λ with respect to the Lebesgues' measure on $\Re^n$. With an additional homogeneity assumption, ρ becomes constant.

We define MPSI by a countable array of random pairs $x_i, M_i, \quad i - 1,2,...,N_1$, possessing the following properties:

x_i re points from the PE;

The random variables M_i are independent, uniformly distributed and independent in the Poissonian ensemble;

The number of points N_1 in any area of finite volume $|V_1|$, the point locations x_i and the parameters characterizing the points M_i are independent random variables.

The ensemble $(x_1;M_1), (x_2;M_2),...,(x_{N_i};M_{N_i})$, is called a regular marked point process and the variables $M_1, M_2,...,M_{N_i}$ are called marks.

Assume that the PE of points is defined in the area $V \subset \Re^n$. It can be also assumed that there are local disturbances in any point of the PE. The points produce "field of influence" based on "law of influence". The influence is called "point source of influence" and the law is called "influence function". In the general case, the influence function is defined as $u(x, y, M)$, where x is the location of the point, y is the location influenced by the point, M and is the intensity of the influence. The quantities x,y and M are vectors.

The main task of the MPSI is the analysis of:

$$\xi = \sum u(x_i, y_i, M_i)$$

where $x_i \in PE$ for i=1,2,…,N_1. So we are interested in characterizing the random field of influences caused by the entire Poissonian ensemble of disturbances. For simplicity, we can assume that the coordinate system's origin is in y, i.e.:

$$\xi = \sum_{x_i \in PE} u(x_i, M_i)$$

The sum field ξ determined by all the PE of random shocks in area V can be analyzed as a boundary of the combined field in the increasing series of subareas of V. Denote by S_R the sphere in $\Re^n$ with a center at the origin and radius R and let $V_R = V \cap S_R$. The number of PE points in V_R is N_R and these points generate the field

$$\xi_R = \sum_{i=1}^{N_i} u(x_i, M_i), x_i \in V_R$$

The combined field ξ produced by the whole PE is defined as the weak limit

$$\xi = \lim_{R \to \infty} \xi_R$$

To show the existence of the above limit, it is enough to check the limiting behavior as $R \to \infty$ of the characteristic function

$$\chi_R(t) E \exp(i(t, \xi_R)), t \in \Re^n$$

see [44], where general conditions for the existence of the limit are derived.

Recall now that the most important features of the MPSI are the following two facts:

The PE of disturbances; and the functions of influence $u(x, M)$ determine the disturbances.

We should emphasize that the PE leads to a Poissonian-summation scheme that guarantees the infinite partitioning of the random variable ξ. The functions of influence $u(x, M)$ are used only for definition of the measure $\mu(A) = \rho \int_{A^*} P(dM) dx$, where A is Borel subset of $\overline{V} \in \Re^n$ and $A^* = \{(x, M) : u(x; M) \in A\}$. If we use point sources of influence that do not possess Poissonian distribution, the corresponding results will be quite different from ours.

STABLE RANDOM FIELDS

Consider the $2D$ plane with points defined by the coordinates t_1 and t_2. A characteristic x is a function of the space coordinates t. If $x(t)$ is a random function it can be considered as a $2D$-random field. The change of the considered random function along a straight line will form a random process that can be called a section of the random field. However, it will no longer be of the type of the model .

The random field is determined by the distribution of the random field values $x(t)$ in n any points of the considered area $x(t_1), x(t_2),...,x(t_t)$. If this distribution is multivariable stable, it is possible to refer the field as a stable field, see [35]. The model is applicable in this case with little modifications for the memory function.

In the most general case the stable fields are heterogeneous and highly anisotropic (the variance among sections of the random field is very high), i.e. the field section properties depend on not only of the location but also on the direction. But there are also isotropic fields (sections of the random field are independent on the direction). An isotropic homogeneous field with a section defined by α-stable motion can be defined as an

α-stable Levy field, determined by the index of stability α and the scale parameters w or γ.

For illustration coincide the Brownian motion of particles in $2D$-constant gradient field. Along a line perpendicular to the direction of the gradient we can count the number of particles passing through a series of line intervals with a constant length. If the Brownian motion was absent (only a gradient flow) the particle density distribution is uniform. As a result of the Brownian motion, the particle density distribution becomes Gaussian. When the process is defined the point sources of influence, depending on the parameters, the particle density will deviate from the Gaussian model and often becomes closer to an α-stable process; in some intervals the particle density can reach very large values - a typical picture of -stable density.

STUDY OF THE SCALAR FIELD OBTAINED BY THE MODEL OF THE POINT SOURCE OF INFLUENCE

The computer program, implementing the proposed algorithm, outputs data for the local potential field for each time step of the particle movement. Using this information we can create *3-D* kriging models. Very important is the problem of creating three-dimensional maps. In most cases the maps are plotted on the base of regular grid of measurements; in our case the measurements are located randomly. So we have to define a regular grid within our area of study and to interpolate the values in the grid points using our randomly located measurements. The procedure is based on the inverse distance method using the Euclidean D_{ik} distances between the interpolation (k) and observation () points:

$$Y_k = \frac{\sum_{i=1}^{n} \left(Y_i / D_{ik}^{a}\right)}{\sum_{i=1}^{n} \left(1 / D_{ik}^{a}\right)},$$

where Y_k is the interpolated value at the k th grid point, Y_i is the measured value in the i-th observation point, α is equal to 1 or 2.

In the literature there are many versions and modifications of the weight function. For example, the so-called J. Matheron geostatistical method [23] [24] uses for this purpose the variogram function, which is similar to the structure function defined by Kolmogorov [20]. The common point among all methods is the determination of weighted average.

In some cases we can attempt to solve the direct problem, in other --- the reverse problem. It is obvious that for the reverse problem, the determining average value is a bad function. In our opinion, this approach has serious drawbacks, although there are opposite opinions (cf. [29], [11], [13] and [42]). On the other hand, in many cases the original data seems to follow the stable distribution and, as it was mention above, this is a plausible model for processes with point sources of influence.

METHOD OF EVALUATING THE VECTOR FIELD

Apparently, the already mentioned inverse problem involves the evaluation of the potential at a random point using discrete measurements of the field. With real data, the problem of dispersion is of great importance. Typically, the range of the geo-chemical data is within $10^{-7} \div 10^{0}$, which in fact implies almost infinite variation. Log transformation of the data significantly reduces the variation.

The analysis of the values shows that for most of the chemical elements the expected value and value of γ differ significantly. This difference can produce significant errors when we analyze anomaly areas characterized with higher or lower concentrations.

The proposed method for data processing is very simple and gives reliable results. In the general case, the method procedure includes the following steps:

Evaluation of the parameters α and γ for the characteristic function of symmetric α-stable ($S\alpha S$) distribution using sets with limited number of observations (< 100). The applied method is an optimized version of the method of Press [4].

Determination of the correlation function of $S\alpha S$ process using the data along a section of space.

Development of modified version of *3D* kriging simulation based on a weight function $w(r)$ defined as $w(r) = 1/\|D(x,y)\|^{C_{yar}}$, $\sum w(r) = 1$, where r is the correlation function of $S\alpha S$ of process, $\|D\|$ is distance between a y_i, x_i pairs, and C is a scale dependent constant. Optimizing weight coefficients $w(r)$ stability to α.

Simulation of *3D* $S\alpha S$ space data using the method of Point Source of Influence (MPSI) proposed by Zolatorev:

Poassonian of points x_i with λ density is generated in the simulation volume. The existing permeability observations are included in the assemble by means of Poassonian density screening [10];

1. For each point x_i $S\alpha S$ random values M_i with γ mean are generated;
2. For each point x_i an influence function is defined $u = 1/\|D(x,y)\|^{C_{yar}}$ where r is the correlation function of $S\alpha S$ process;
3. For each point y_i (located on structured or unstructured grid within the simulation volume) the estimated value z_i is defined as $z_i = \sum_{x_i} u(x_i, y_i, M_i)$.
4. Check stability of α in original and simulation data

The pilot outcomes three dimension Alpha Stable Kriging are shown accordingly on Fig. 1, 2, 3. On the first figure is shown model, which is founded on reference tools of three-dimensional simulation, in a case utilized of possibilities of the program Rock Works. is it's visible that the model is very chaotic. On next figures is shown model on a basis Alpha Stable Kriging. The model is very well compounded with the substantial geological data, which is compounded with outcomes shown on next Figure (3). Cross Validation shows, that Alpha Stable Kriging is a successful formalism for an estimate of a mining deposit.

CONCLUSION

The obtained results have methodological importance and give the basis for development of the method, so it can allow us to develop better model representations of complex natural processes.

The method application to difficult equations of a gradient will require further researches, to take into account processes, arising in *3D* models, in particular occurrences of forces of interaction.

The results obtained in this paper allow us to assume, that the inclusion parameterization of geological processes will not render of influence to adequacy of application base MPSI. However, a more detailed research on this problem within the framework of complete stochastic models is required.

Use of this engineering for construction of common model MPSI is represented perspective, though already in zero approach there are questions, on which at present there is no answer.

The proposed method can be considered as a new technique for modeling and evaluation of processes with heavy tailed distributions.

The computer experiments show that the method can be applied in many areas of modeling real phenomena --- physics, geology, environmental studies, hydrology, etc.

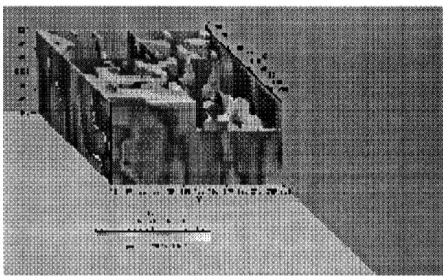

Figure 1. Here is shown 3D model on base Inverse distance method. It is visible, that the geological object (Mining deposit Kremikovtci) is rather random

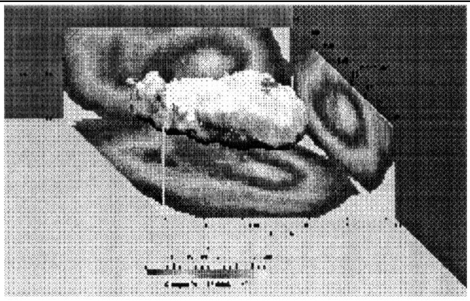

Figure 2. Here is shown 3D model on base alpha stable kriging method. It is visible, that the geological object (Mining deposit Kremikovtci) is rather compact and the zones such as " of Mining poles " are planned

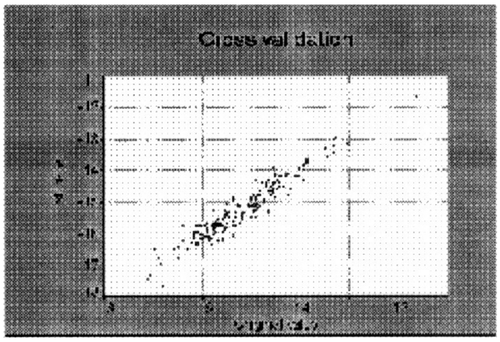

Figure 3. Here is shown cross validation matching between the original data (horizontal axes) and estimation in the same points on base alpha stable kriging method (vertical axes). It is visible, that the correlation very good, that is model has no a problem from calibration..

The proposed approach for solution of the inverse problem is at initial level of development, as there are no additional tests of its applicability. For now, it is tested and verified only for geostatistical data. The approach effectiveness is due to the new formalism for calculation of the correlation distance among the observations. In the classical geostatistics the solution is obtained using the classical variogram. Except for the Stable modification of the variogram for the Gaussian model, all other variogram models are useless. This conclusion is very important for the methods of image processing and map generation.

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