ON THE NEUTRAL LOSSLESS TRANSMISSION LINE EQUATIONS WITH TUNNEL DIODE AND A LUMPED PARALLEL CAPACITANCE

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ABSTRACT

By means of a fixed point approach an existence theorem for neutral equations arising in transmission lines is proved.

The present paper continues investigations on lossless transmission lines equations with nonlinear resistive elements.

In a previous paper [1] we have studied neutral equations (2)-(4) with nonlinearities caused by a nonlinear \( V \) characteristic which is of polynomial type. Here we consider an initial value problem for neutral equations with an exponential nonlinearity in the right-hand side. Such a problem is equivalent (cf. [5]-[7], [2]) to an initial-boundary value problem for linear hyperbolic system with a nonlinear boundary conditions. Indeed in some cases the approximation of the \( V-I \) characteristic curve generated by the diffusion current in semiconductors [8]-[9] can be fitted by an exponential function. The relation between the diffusion current and the voltage is of the type \( i = \alpha \cdot e^\gamma u \), where \( \alpha \) and \( \gamma \) are constants. It is known that such type nonlinearities have also another applications (cf. [10], [11]).

Therefore formulating the initial value problem

\[ u(t) = F(u(\Delta_1(t)), \ldots, u(\Delta_m(t)), \dot{u}(\gamma_1(t)), \ldots, \dot{u}(\gamma_n(t))), \quad t > 0 \]

\[ u(t) = \phi(t), \quad t \leq 0 \]

\[ u(t) = \dot{\phi}(t), \quad t \leq 0, \]  

we have to impose such conditions on the right-hand side of (1), \( F(u_1, \ldots, u_m, \dot{v}_1, \ldots, \dot{v}_n) \), in order to include exponential nonlinearities. For instance \( F \) can be chosen of the type

\[ F(u_1, \ldots, u_m, \dot{v}_1, \ldots, \dot{v}_n) = A \sum_{k=1}^{n} \left( e^{\alpha_k u_k} - 1 \right) + B \sum_{s=1}^{m} \dot{v}_s. \]

We reduce the problem (1) to the following one (putting \( \gamma(t) = \dot{u}(t) \) for \( t > 0 \) and \( \psi(t) = \dot{\phi}(t) \) for \( t \leq 0 \) assuming \( \phi(0) = 0 \)):

\[ y(t) = F \left( \int_0^t y(\sigma)d\sigma, \ldots, \int_0^t y(\gamma(t))d\gamma(t), \ldots, y(\gamma_n(t)) \right), \quad t \in [0, T_0] \]

\[ y(t) = \psi(t), \quad t \leq 0. \]  

We make the following assumptions (C):

(C1) \( \Delta_i(t), \gamma_s(t) : R^1 \rightarrow R^1 \)

\( i = 1, \ldots, m; s = 1, \ldots, n \) are continuous and \( t - \Delta_i(t) \geq \Delta_0 > 0 \), \( t - \gamma_s(t) \geq \gamma_0 > 0 \) for some constants \( \Delta_0, \gamma_0 \).

(C2) the functions \( F(u_1, \ldots, u_m, v_1, \ldots, v_n) : R^{m+n} \rightarrow R^1 \) and \( \psi() : R^1 \rightarrow R^1 \) are continuous and satisfies the condition

\[ \psi(0) = F \left( \int_0^\Delta \psi(s)ds, \ldots, \int_0^\Delta \psi(\gamma(t))ds, \ldots, \psi(\gamma_n(t)) \right) = F(0, \ldots, 0, \psi(0), \ldots, \psi(0)). \]

(C3) \[ |F(u_1, \ldots, u_m, v_1, \ldots, v_n)| \leq A \sum_{k=1}^{n} e^{\alpha_k u_k - 1} + B \sum_{s=1}^{m} |\dot{v}_s| \]

where \( A, B \) are positive constants.

(C4) \[ |F(u_1, \ldots, u_m, v_1, \ldots, v_n) - F(\bar{u}_1, \bar{u}_2, \ldots, \bar{u}_m, \bar{v}_1, \ldots, \bar{v}_n)| \leq A \sum_{k=1}^{m} \left| e^{\alpha_k u_k} - e^{\alpha_k \bar{u}_k} \right| + B \sum_{s=1}^{n} |\dot{v}_s - \dot{\bar{v}}_s|, \]

where \( A_1 \) and \( B_1 \) are positive constants.

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This document contains mathematical text and proofs. It discusses solutions to initial value problems and introduces a subset of functions with specific properties. The text is related to the study of lossless transmission and involves the use of continuous solutions and operators. The notation and formulas are typical of advanced mathematics, particularly in the field of differential equations and operator theory.
\[ \leq e^{\lambda} p_{\lambda}(f, \tilde{f}) \leq \left[ A_1 e^{-\lambda \Delta_0} e^{\lambda T_0} \sum_{k=1}^{m} |\alpha_k| + B_1 e^{-\lambda \gamma_0} \right]. \]

Consequently

\[ \rho_{\lambda}(TF, TF) \leq \left[ A_1 \sum_{k=1}^{m} |\alpha_k| e^{\lambda T_0 - \lambda \Delta_0} \right] + nB_1 e^{-\lambda \gamma_0}. \]

But for sufficiently large \( \lambda \), the expression in the bracket is smaller than 1 (if \( T_0 - \Delta_0 \leq 0 \)) and therefore \( T \) is contractive operator.

Finally we have to choose an initial approximation. Indeed let

\[ x_0(t) = \begin{cases} 
\psi(t), & t \leq 0 \\
\psi(0), & t > 0
\end{cases}. \]

Then

\[ |(TFx_0)(t) - x_0(t)| = \begin{cases} 
0, & t \leq 0 \\
|F(0, \ldots, 0, \psi(0), \ldots, \psi(0))|, & t > 0
\end{cases}. \]

The map \( j : A \rightarrow A \) in this case is \( j(\lambda) = \lambda \Rightarrow j^k(\lambda) = \lambda \), i.e.

\[ \rho_{j^k(\lambda)}(x_0, T x_0) \leq \infty (k = 1, 2, \ldots). \]

Therefore \( T \) has a unique fixed point which a solution of (2).

Theorem is thus proved.

REFERENCES


